General relativity in ordinary three-dimensional space

David B. Parker*
pgu.org
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If you start with general relativity and you keep Einstein's field equation, but you replace the geodesic equation with a particular non-covariant equation for a curved line in flat space, then you get a theory of gravity that makes the same physical predictions as general relativity, except that space is ordinary flat three-dimensional cartesian space and time is ordinary absolute universal time.

If you start with general relativity and you keep Einstein's field equation[1], but you replace the geodesic equation:

$$\frac{\mathrm{d}^2 x^\alpha}{\mathrm{d}\tau^2} = -\Gamma^\alpha_{\beta\gamma}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}\frac{\mathrm{d}x^\gamma}{\mathrm{d}\tau} \tag{1}$$

with the following non-covariant equation for a curved line in flat space:

$$\begin{aligned}\frac{\mathrm{d}^2 x^i}{\mathrm{d}(x^0)^2} = &-\left(\Gamma^i_{00} + 2\Gamma^i_{j0}\frac{\mathrm{d}x^j}{\mathrm{d}x^0} + \Gamma^i_{jk}\frac{\mathrm{d}x^j}{\mathrm{d}x^0}\frac{\mathrm{d}x^k}{\mathrm{d}x^0}\right)\\ &+\frac{\mathrm{d}x^i}{\mathrm{d}x^0}\left(\Gamma^0_{00} + 2\Gamma^0_{j0}\frac{\mathrm{d}x^j}{\mathrm{d}x^0} + \Gamma^0_{jk}\frac{\mathrm{d}x^j}{\mathrm{d}x^0}\frac{\mathrm{d}x^k}{\mathrm{d}x^0}\right),\end{aligned} \tag{2}$$

then you get a non-covariant theory of gravity[2][3][4] that makes the same physical predictions as general relativity, except that space is ordinary flat three-dimensional cartesian space ($x^1, x^2, x^3 = x, y, z$) and time is ordinary absolute universal time ($x^0 = ct$).

I have just learned, while double-checking this letter, that equation (2) appears almost exactly in [5] as an intermediate step in the derivation of the post-newtonian approximation, which means it may also appear in other peoples' works. After deriving equation (2), rather that use it as-is, Weinberg goes on to expand it in inverse powers of the speed of light to get a better approximation to gravitation in a system of slowly moving particles.

I am not sure if or why Weinberg (and perhaps others) did not recognize that by abandoning general covariance and the metric interpretation of the gravitational field[2], that Einstein's field equation and equation (2) could be the basis for a non-covariant theory of gravity that in ordinary three-dimensional space with absolute time, produces the same physical predictions as general relativity.

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* Electronic address: `daveparker@pgu.org`
[1] Einstein, A., "The Foundation of the General Theory of Relativity", 1916, pgs 111-164, translated by Perrett, W., and Jeffery G. B.,
[2] Parker, D. B., "General relativity, general covariance, and absolute gravity", 2023, preprint, https://pgu.org
[3] Parker, D. B., "On absolute clocks and rulers", 2023, preprint, https://pgu.org
[4] Parker, D. B., "General Relativity in Absolute Space and Time", 2022, preprint, https://pgu.org
[5] Weinberg, S, Gravitation and Cosmology, 1972, pg 213, eq 9.1.2