Abstract

The Michelson-Morley experiment led Einstein to introduce the concept of spacetime and to propose that all of the laws of physics are Lorentz invariant. The Michelson-Morley experiment has been amply confirmed, giving Einstein's proposal an aura of near certainty. However, I would like to propose a new way to explain the Michelson-Morley experiment that retains the Lorentz invariance of Maxwell's equations without requiring the other laws of physics to be Lorentz invariant. In this new theory, Lorentz invariance is not fundamental, but instead is a consequence of the fact that Maxwell's equations are incomplete because they lack a way to define local inertial reference frames. This new theory explicitly defines 3-dimensional local inertial reference frames in terms of the gravitational potential V_G along with a momentum potential $\mathbf{V_P}$ and a force potential $\mathbf{V_F}$. This new theory decouples space and time, and explains the Michelson-Morley experiment in ordinary 3-dimensional space.

How to explain the Michelson-Morley experiment in ordinary 3-dimensional space

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1 Introduction

The Michelson-Morley experiment led Einstein to introduce the concept of spacetime and to propose that all of the laws of physics are Lorentz invariant. The Michelson-Morley experiment has been amply confirmed, giving Einstein's proposal an aura of near certainty[5].

However, I would like to propose a new way to explain the Michelson-Morley experiment that retains the Lorentz invariance of Maxwell's equations without requiring the other laws of physics to be Lorentz invariant. In this new theory, Lorentz invariance is not fundamental, but instead is a consequence of the fact that Maxwell's equations are incomplete because they lack a way to define local inertial reference frames.

This new theory explicitly defines 3-dimensional local inertial reference frames in terms of the gravitational potential V_G along with a momentum potential $\mathbf{V_P}$ and a force potential $\mathbf{V_F}$. This new theory decouples space and time, and explains the Michelson-Morley experiment in ordinary 3-dimensional space.

To visualize how the V_G , $\mathbf{V_P}$, and $\mathbf{V_F}$ potentials define local inertial reference frames, imagine that the universe has an absolute 3-dimensional Euclidean coordinate system and an absolute time that proceeds at the same rate everywhere. Imagine a physicist on the surface of the Earth. Imagine a photon near the physicist, and imagine that the photon wants to modify its speed in absolute space so that its speed relative to the physicist is c in all directions (which will also have the effect of modifying the physics of the physicist's clocks so that they measure local time instead of absolute time).

The gradient of the gravitational potential, ∇V_G , forms a vector field pointing towards the center of the Earth. However, ∇V_G alone does not provide enough information for the photon to adjust its speed relative to the physicist on the rotating Earth. For example, the photon cannot determine how fast the Earth is rotating.

The momentum potential $\mathbf{V}_{\mathbf{P}}$ adds a vector field circulating around the Earth. The force potential $\mathbf{V}_{\mathbf{F}}$ adds a vector field pointing towards the axis of the Earth's rotation. Together ∇V_G , $\mathbf{V}_{\mathbf{P}}$, and $\mathbf{V}_{\mathbf{F}}$ form a non-orthogonal 3-dimensional planetary coordinate system around the Earth. The planetary coordinate system provides enough information for the photon to calculate a local inertial reference frame for itself.

2 The Momentum and Force Potentials Around a Massive Rotating Sphere

The gravitational potential V_G is a scalar potential, and the momentum potential $\mathbf{V}_{\mathbf{P}}$ and force potential $\mathbf{V}_{\mathbf{F}}$ are vector potentials. $\mathbf{V}_{\mathbf{P}}$ and $\mathbf{V}_{\mathbf{F}}$ are calculated in the same way as V_G :

$$V_G(\mathbf{s}) = \int_V \frac{\sigma_{\mathbf{G}}(\mathbf{s}')}{|\mathbf{s} - \mathbf{s}'|} \, \mathrm{d}V$$
$$\mathbf{V}_{\mathbf{P}}(\mathbf{s}) = \int_V \frac{\sigma_{\mathbf{P}}(\mathbf{s}')}{|\mathbf{s} - \mathbf{s}'|} \, \mathrm{d}V$$
$$\mathbf{V}_{\mathbf{F}}(\mathbf{s}) = \int_V \frac{\sigma_{\mathbf{F}}(\mathbf{s}')}{|\mathbf{s} - \mathbf{s}'|} \, \mathrm{d}V$$

where $\sigma_{\mathbf{G}}$ is the scalar mass density, $\boldsymbol{\sigma}_{\mathbf{P}}$ is the vector momentum density (e.g. mass density times velocity), $\boldsymbol{\sigma}_{\mathbf{F}}$ is the vector force density (e.g. mass density times acceleration), \mathbf{s} is the point where we are calculating the potential, \mathbf{s}' is a point in the volume of integration, and V is the volume of integration.

In this paper we will use (ρ, θ, ϕ) as spherical coordinates and (r, θ, z) as cylindrical coordinates. The θ coordinate is the same in both cases.

The potentials V_G , $\mathbf{V_P}$, and $\mathbf{V_F}$ at a point (ρ, θ, ϕ) outside of a uniformlydense massive rotating sphere whose center is at the origin is:

$$V_G(\rho, \theta, \phi) = \frac{M}{\rho}$$
$$\mathbf{V}_{\mathbf{P}}(\rho, \theta, \phi) = \frac{\omega \sin \phi \, MR^2}{5 \, \rho^2} \hat{\theta}$$
$$\mathbf{V}_{\mathbf{F}}(\rho, \theta, \phi) = \frac{-\omega^2 \sin \phi \, MR^2}{5 \, \rho^2} \hat{\mathbf{r}}$$

where M is the mass of the sphere, R is the radius of the sphere, and $\omega = d\theta/dt$ is the angular rotation speed of the sphere around the z axis. The gradient of the gravitational potential, ∇V_G , is:

$$\nabla V_G(\rho, \theta, \phi) = \frac{-M}{\rho^2} \hat{\rho}$$

 ∇V_G , $\mathbf{V_P}$, and $\mathbf{V_F}$ form a non-orthogonal 3-dimensional planetary coordinate system around the massive rotating sphere. ∇V_G points toward the center of the sphere, $\mathbf{V_P}$ circulates around the sphere, and $\mathbf{V_F}$ points toward the axis of rotation. The planetary coordinate system is a hybrid of spherical and cylindrical coordinates. For example, the magnitude of $\mathbf{V_F}$ is easiest to express in spherical coordinates, while its direction is easiest to express in terms of the cylindrical unit vector $\hat{\mathbf{r}} = \sin \phi \, \hat{\boldsymbol{\rho}} + \cos \phi \, \hat{\boldsymbol{\phi}}$.

If the rotating sphere is moving through absolute space with a constant velocity \mathbf{v} , we can still keep the origin of the coordinate system at the center of the sphere. The only difference is that the momentum potential $\mathbf{V}_{\mathbf{P}}$ acquires an additional term for the sphere's total linear momentum:

$$\mathbf{V}_{\mathbf{P}}(\rho,\theta,\phi) = \frac{\omega \sin \phi \, M R^2}{5 \, \rho^2} \, \hat{\boldsymbol{\theta}} + \frac{M}{\rho} \, \mathbf{v}$$

The coordinate system used to express the velocity \mathbf{v} is irrelevant because the velocity is constant and so it will disappear when we take the derivatives in the curl $\nabla \times (\mathbf{V}_{\mathbf{P}}/V_G)$ while calculating a local inertial reference frame:

$$\nabla \times (\mathbf{V}_{\mathbf{P}}/V_G) = \nabla \times (\frac{\frac{\omega \sin \phi \, MR^2}{5 \rho^2} \,\hat{\boldsymbol{\theta}} + \frac{M}{\rho} \, \mathbf{v}}{\frac{M}{\rho}})$$
$$= \nabla \times (\frac{\omega \sin \phi \, R^2}{5 \rho} \,\hat{\boldsymbol{\theta}} + \mathbf{v})$$
$$= \nabla \times (\frac{\omega \sin \phi \, R^2}{5 \rho} \,\hat{\boldsymbol{\theta}}) + \nabla \times \mathbf{v}$$
$$= \nabla \times (\frac{\omega \sin \phi \, R^2}{5 \rho} \,\hat{\boldsymbol{\theta}})$$
$$= \frac{2 \, \omega \cos \phi \, R^2}{5 \, \rho^2} \,\hat{\boldsymbol{\rho}}$$

The similar curl involving V_F that we will use is:

$$\nabla \times (\mathbf{V}_{\mathbf{F}}/V_G) = \frac{2\,\omega^2\,\sin\phi\,\cos\phi\,R^2}{5\,\rho^2}\,\hat{\boldsymbol{\theta}}$$

3 Calculating a Local Inertial Reference Frame

The photon near the physicist can use V_G , $\mathbf{V_P}$, and $\mathbf{V_F}$ to calculate an explicit local inertial reference frame for itself in ordinary 3-dimensional space.

The photon needs three unit vectors for its local inertial reference frame. For convenience, we will choose them to be the spherical unit vectors $\hat{\rho}$, $\hat{\theta}$, and $\hat{\phi}$. In terms of V_G , $\mathbf{V_P}$, and $\mathbf{V_F}$, one way to calculate the spherical unit vectors is:

$$\hat{\boldsymbol{
ho}} = rac{-
abla V_G}{|
abla V_G|}, \quad \hat{\boldsymbol{ heta}} = \hat{\boldsymbol{
ho}} imes rac{-\mathbf{V_F}}{|\mathbf{V_F}|} \quad \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{ heta}} imes \hat{\boldsymbol{
ho}}$$

The three other quantities the photon needs are its distance ρ from the center of the Earth, its angle ϕ from the z axis (i.e. its latitude), and the local inertial reference frame's angular speed $d\theta/dt$ around the z axis. One way to calculate those quantities in terms of V_G , $\mathbf{V_P}$, and $\mathbf{V_F}$ is:

$$\rho = \frac{V_G}{|\nabla V_G|}, \quad \phi = \tan^{-1} \left(\frac{\mathbf{V}_{\mathbf{F}} \cdot \hat{\boldsymbol{\rho}}}{\mathbf{V}_{\mathbf{F}} \cdot \hat{\boldsymbol{\phi}}} \right) \quad \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{|\nabla \times (\mathbf{V}_{\mathbf{F}}/V_G)|}{\sin \phi |\nabla \times (\mathbf{V}_{\mathbf{P}}/V_G)|},$$

The photon now has all the information it needs in order to modify its speed in absolute space so that its speed relative to the physicist on the rotating Earth is c in all directions, thus explaining the Michelson-Morley experiment in ordinary 3-dimensional space.

4 Technical Details

A technical difficulty with calculating $\mathbf{V}_{\mathbf{F}}$ and $\mathbf{V}_{\mathbf{F}}$ around a massive rotating sphere is that the definite integrals are elliptical. None of the symbolic math packages I tried could do them. This section shows a way to do them by hand, by rotating the problem to remove the elliptical integrals.

We will demonstrate the technique using $V_{\mathbf{P}}$. The same method works for $V_{\mathbf{F}}$. We will start with the formula for $V_{\mathbf{P}}$:

$$\mathbf{V}_{\mathbf{P}}(\mathbf{s}) = \int_{V} \frac{\boldsymbol{\sigma}_{\mathbf{P}}(\mathbf{s}')}{|\mathbf{s} - \mathbf{s}'|} \, \mathrm{d}V$$

A natural way to set up the calculation of $\mathbf{V}_{\mathbf{P}}$ around a massive rotating sphere is in cylindrical coordinates. To keep the calculations as simple as possible we will assume that the sphere has a uniform mass density $\sigma_{\mathbf{G}} = 3M/(4\pi R^3)$. The velocity at a point (r, θ, z) inside a sphere rotating with an angular speed ω around the z axis is $\omega r \hat{\theta}$. Letting the momentum density $\sigma_{\mathbf{P}}$ be the mass density times the velocity gives:

$$\boldsymbol{\sigma}_{\mathbf{P}}(r, \theta, z) = \sigma_{\mathbf{G}} \,\omega \, r \, \hat{\boldsymbol{\theta}}$$

The unit vector $\hat{\boldsymbol{\theta}}$ is the same in spherical coordinates as in cylindrical, so we can convert the momentum density to spherical coordinates by making the substitution $r = \rho \sin \phi$:

$$\boldsymbol{\sigma}_{\mathbf{P}}(\rho,\theta,\phi) = \sigma_{\mathbf{G}}\,\omega\rho\sin\phi\,\hat{\boldsymbol{\theta}}$$

We will let the initial point of integration **s** be $(\rho_0, \theta_0, \phi_0)$:

$$\mathbf{V}_{\mathbf{P}}(\mathbf{s}) = \mathbf{V}_{\mathbf{P}}(\rho_0, \theta_0, \phi_0)$$

To eliminate the elliptical integrals we are now going to rotate the problem by $-\theta_0$ around the z axis and then by $-\phi_0$ around the y axis. That will tilt the sphere's axis of rotation off of the z axis and put the point of integration on the z axis. The rotated point of integration is:

$$\mathbf{V}_{\mathbf{P}}(\mathbf{s}) = \mathbf{V}_{\mathbf{P}}(\rho_0, 0, 0)$$

To rotate $\sigma_{\mathbf{P}}$ we will first convert it from spherical to cartesian coordinates using the substitutions:

$$\rho = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1}(y, x), \quad \phi = \cos^{-1}(z/\rho)$$
$$\hat{\rho} = \cos\theta \sin\phi \,\hat{\mathbf{x}} + \sin\theta \sin\phi \,\hat{\mathbf{y}} + \cos\phi \,\hat{\mathbf{z}}$$
$$\hat{\theta} = -\sin\theta \,\hat{\mathbf{x}} + \cos\theta \,\hat{\mathbf{y}}$$
$$\hat{\phi} = \cos\theta \cos\phi \,\hat{\mathbf{x}} + \sin\theta \cos\phi \,\hat{\mathbf{y}} - \sin\phi \,\hat{\mathbf{z}}$$

where θ goes from 0 to 2π , ϕ goes from 0 to π , $\sin \phi = \sqrt{x^2 + y^2}/\rho$, and where the two-argument form of \tan^{-1} indicates that the quadrant of xand y determines the value of θ so that $\sin \theta = y/\sqrt{x^2 + y^2}$ and $\cos \theta = x/\sqrt{x^2 + y^2}$.

After substituting, $\sigma_{\rm P}$ in cartesian coordinates simplifies to:

$$\boldsymbol{\sigma}_{\mathbf{P}}(x, y, z) = \boldsymbol{\sigma}_{\mathbf{G}} \,\omega \left(-y \,\hat{\mathbf{x}} + x \,\hat{\mathbf{y}}\right)$$

The matrix **R** that rotates points by $-\theta_0$ around the *z* axis and then by $-\phi_0$ around the *y* axis is the product of the two rotation matrices **R** = **R** $(-\phi_0)$ **R** $(-\theta_0)$. In order to find substitutions for the unrotated coordinates in terms of the rotated coordinates, we need the inverse of that matrix, $\mathbf{R}^{-1} = \mathbf{R}(\theta_0)\mathbf{R}(\phi_0)$:

$$\mathbf{R}^{-1} = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 & 0\\ \sin\theta_0 & \cos\theta_0 & 0\\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi_0 & 0 & \sin\phi_0\\ 0 & 1 & 0\\ -\sin\phi_0 & 0 & \cos\phi_0 \end{bmatrix}$$
$$= \begin{bmatrix} \cos\theta_0\cos\phi_0 & -\sin\theta_0 & \cos\theta_0\sin\phi_0\\ \sin\theta_0\cos\phi_0 & \cos\theta_0 & \sin\theta_0\sin\phi_0\\ -\sin\phi_0 & 0 & \cos\phi_0 \end{bmatrix}$$

The values to substitute for $x, y, z, \hat{\mathbf{x}}, \hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are then:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R}^{-1} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \theta_0 \cos \phi_0 x' - \sin \theta_0 y' + \cos \theta_0 \sin \phi_0 z' \\ \sin \theta_0 \cos \phi_0 x' + \cos \theta_0 y' + \sin \theta_0 \sin \phi_0 z' \\ -\sin \phi_0 x' + \cos \phi_0 z' \end{bmatrix}$$

$$\begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{y}} \\ \hat{\mathbf{z}} \end{bmatrix} = \mathbf{R}^{-1} \begin{bmatrix} \hat{\mathbf{x}}' \\ \hat{\mathbf{y}}' \\ \hat{\mathbf{z}}' \end{bmatrix} = \begin{bmatrix} \cos\theta_0 \cos\phi_0 \,\hat{\mathbf{x}}' - \sin\theta_0 \,\hat{\mathbf{y}}' + \cos\theta_0 \sin\phi_0 \,\hat{\mathbf{z}}' \\ \sin\theta_0 \cos\phi_0 \,\hat{\mathbf{x}}' + \cos\theta_0 \,\hat{\mathbf{y}}' + \sin\theta_0 \sin\phi_0 \,\hat{\mathbf{z}}' \\ -\sin\phi_0 \,\hat{\mathbf{x}}' + \cos\phi_0 \,\hat{\mathbf{z}}' \end{bmatrix}$$

Substituting and simplifying, the rotated $\sigma_{\rm P}$ in cartesian coordinates is:

$$\boldsymbol{\sigma}_{\mathbf{P}}(x',y',z') = \boldsymbol{\sigma}_{\mathbf{G}}\,\omega\left(-\cos\phi_0\,y'\,\hat{\mathbf{x}}' + (\cos\phi_0\,x' + \sin\phi_0\,z')\,\hat{\mathbf{y}}' - \sin\phi_0\,y'\,\hat{\mathbf{z}}'\right)$$

Notice that there are no terms in θ_0 . We could have invoked rotational symmetry earlier in order to ignore the rotation by $-\theta_0$.

The integrals will be easier to evaluate if we now convert the problem back to spherical coordinates using the substitutions:

$$\begin{aligned} x' &= \rho' \cos \theta' \sin \phi', \quad y' &= \rho' \sin \theta' \sin \phi', \quad z' &= \rho' \cos \phi' \\ \hat{\mathbf{x}}' &= \cos \theta' \sin \phi' \,\hat{\boldsymbol{\rho}'} - \sin \theta' \,\hat{\boldsymbol{\theta}'} + \cos \theta' \cos \phi' \,\hat{\boldsymbol{\phi}'} \\ \hat{\mathbf{y}}' &= \sin \theta' \sin \phi' \,\hat{\boldsymbol{\rho}'} + \cos \theta' \,\hat{\boldsymbol{\theta}'} + \sin \theta' \cos \phi' \,\hat{\boldsymbol{\phi}'} \\ \hat{\mathbf{z}}' &= \cos \phi' \,\hat{\boldsymbol{\rho}'} - \sin \phi' \,\hat{\boldsymbol{\phi}'} \end{aligned}$$

Substituting and simplifying, the rotated $\sigma_{\rm P}$ in spherical coordinates is:

$$\sigma_{\mathbf{P}}(\rho',\theta',\phi') = \sigma_{\mathbf{G}}\,\omega\,\rho'\,(\,(\cos\phi_0\sin\phi'+\sin\phi_0\cos\theta'\cos\phi')\,\hat{\theta'}+\sin\phi_0\sin\theta'\,\hat{\phi'}\,)$$

We will also need the distance $|\mathbf{s} - \mathbf{s}'|$ from the point \mathbf{s} at $(\rho_0, 0, 0)$ to the point \mathbf{s}' at (ρ', θ', ϕ') :

$$|\mathbf{s} - \mathbf{s}'| = \sqrt{\rho_0^2 + \rho'^2 - 2\rho_0 \rho' \cos \phi'}$$

The problem is now non-elliptical. Substituting for $\sigma_{\mathbf{P}}$ and $|\mathbf{s} - \mathbf{s}'|$ in the equation for $\mathbf{V}_{\mathbf{P}}$, then separating the integrals involving $\hat{\theta'}$ and $\hat{\phi'}$ gives:

$$\begin{aligned} \mathbf{V}_{\mathbf{P}}(\rho_0, 0, 0) &= \sigma_{\mathbf{G}} \,\omega \int_V \frac{\rho'(\cos \phi_0 \sin \phi' + \sin \phi_0 \cos \theta' \cos \phi')}{\sqrt{\rho_0^2 + \rho'^2 - 2\rho_0 \rho' \cos \phi'}} \,\hat{\boldsymbol{\theta}'} \,\mathrm{d}V \\ &+ \sigma_{\mathbf{G}} \,\omega \int_V \frac{\rho'_0 \sin \theta' \sin \phi_0}{\sqrt{\rho_0^2 + \rho'^2 - 2\rho_0 \rho' \cos \phi'}} \,\hat{\boldsymbol{\phi}'} \,\mathrm{d}V \end{aligned}$$

The unit vectors $\hat{\rho'}$, $\hat{\theta'}$, and $\hat{\phi'}$ at (ρ', θ', ϕ') in terms of the unit vectors $\hat{\rho}$, $\hat{\theta}$, and $\hat{\phi}$ at $(\rho_0, 0, 0)$ are:

$$\hat{\rho'} = \cos \phi' \,\hat{\rho} + \sin \theta' \sin \phi' \,\hat{\theta} + \cos \theta' \sin \phi' \,\hat{\phi}$$
$$\hat{\theta'} = \cos \theta' \,\hat{\theta} - \sin \theta' \,\hat{\phi}$$
$$\hat{\phi'} = -\sin \phi' \,\hat{\rho} + \sin \theta' \cos \phi' \,\hat{\theta} + \cos \theta' \cos \phi' \,\hat{\phi}$$

Substituting for $\hat{\theta'}$ and $\hat{\phi'}$, and then removing $\hat{\rho}$, $\hat{\theta}$, and $\hat{\phi}$ from inside the integrals gives:

$$\begin{aligned} \mathbf{V}_{\mathbf{P}}(\rho_0, 0, 0) &= -\sigma_{\mathbf{G}}\,\omega\sin\theta_0\,\hat{\boldsymbol{\rho}}\int_V \frac{\rho'\sin\theta'\sin\phi'}{\sqrt{\rho_0^2 + \rho'^2 - 2\rho_0\rho'\cos\phi'}}\,\mathrm{d}V \\ &+ \sigma_{\mathbf{G}}\,\omega\cos\phi_0\,\hat{\boldsymbol{\theta}}\int_V \frac{\rho'\cos\theta'\sin\phi'}{\sqrt{\rho_0^2 + \rho'^2 - 2\rho_0\rho'\cos\phi'}}\,\mathrm{d}V \\ &+ \sigma_{\mathbf{G}}\,\omega\sin\phi_0\,\hat{\boldsymbol{\theta}}\int_V \frac{\rho'\cos\phi'}{\sqrt{\rho_0^2 + \rho'^2 - 2\rho_0\rho'\cos\phi'}}\,\mathrm{d}V \\ &- \sigma_{\mathbf{G}}\,\omega\cos\theta_0\,\hat{\boldsymbol{\phi}}\int_V \frac{\rho'\sin\theta'\sin\phi'}{\sqrt{\rho_0^2 + \rho'^2 - 2\rho_0\rho'\cos\phi'}}\,\mathrm{d}V \end{aligned}$$

When we substitute $dV = \rho'^2 \sin \phi' d\theta' d\phi' d\rho'$ and integrate over the sphere, the integrals involving $\sin \theta'$ or $\cos \theta'$ go to 0 when θ' goes from 0 to 2π , leaving only the integral:

$$\mathbf{V}_{\mathbf{P}}(\rho_{0}, 0, 0) = \sigma_{\mathbf{G}} \,\omega \sin \phi_{0} \,\hat{\boldsymbol{\theta}} \int_{\rho'=0}^{R} \int_{\phi'=0}^{\pi} \int_{\theta'=0}^{2\pi} \frac{\rho'^{3} \cos \phi' \sin \phi' \,\mathrm{d}\theta' \,\mathrm{d}\phi' \,\mathrm{d}\rho'}{\sqrt{\rho_{0}^{2} + \rho'^{2} - 2\rho_{0}\rho' \cos \phi'}}$$

Performing the integration gives:

$$\mathbf{V}_{\mathbf{P}}(\rho_0, 0, 0) = \sigma_{\mathbf{G}} \,\omega \sin \phi_0 \,\hat{\boldsymbol{\theta}} \,\frac{4 \,\pi R^5}{15 \,\rho_0^2}$$

Finally, substituting $\sigma_{\mathbf{G}} = 3M/(4\pi R^3)$ gives:

$$\mathbf{V}_{\mathbf{P}}(\rho_0, 0, 0) = \frac{\omega \sin \phi_0 M R^2}{5 \rho_0^2} \,\hat{\boldsymbol{\theta}}$$

Rotating the problem back to its original orientation changes only the position and orientation of $\hat{\theta}$, leaving the equation for the result unchanged.

To calculate the force potential $\mathbf{V}_{\mathbf{F}}$, it is natural to begin as for the momentum potential $\mathbf{V}_{\mathbf{P}}$ and set up the problem in cylindrical coordinates. The acceleration at a point (r, θ, z) inside a sphere rotating with an angular speed ω around the z axis is $-\omega^2 r \hat{\mathbf{r}}$. Letting the force density $\sigma_{\mathbf{F}}$ be the uniform mass density $\sigma_{\mathbf{G}}$ times the acceleration gives:

$$\boldsymbol{\sigma}_{\mathbf{F}}(r, \theta, z) = -\sigma_{\mathbf{G}} \,\omega^2 \, r \, \hat{\mathbf{r}}$$

Converting $\sigma_{\mathbf{F}}$ to spherical coordinates using the substitutions $r = \rho \sin \phi$ and $\hat{\mathbf{r}} = \sin \phi \, \hat{\boldsymbol{\rho}} + \cos \phi \, \hat{\boldsymbol{\phi}}$ gives:

$$\boldsymbol{\sigma}_{\mathbf{F}}(\rho,\theta,\phi) = -\,\boldsymbol{\sigma}_{\mathbf{G}}\,\omega^2\,\rho\,\sin\phi\,(\sin\phi\,\hat{\boldsymbol{\rho}} + \cos\phi\,\hat{\boldsymbol{\phi}})$$

The calculation of V_F then proceeds along the same lines as the calculation for V_P .

5 Historical Notes

The idea that there might exist a momentum potential based on mass times velocity, analogous to the magnetic potential based on charge times velocity, seems to be an old one that has occurred to many people. I think that Stokes and Lorentz[6] may have investigated something similar in an attempt to resolve surface velocity problems in the aether theories. However, any attempt to use the momentum potential alone to construct an inertial reference frame would have failed because the momentum potential is insufficient without the force potential.

I cannot find any prior reference to the force potential, much less any reference to the idea of using the trio of potentials V_G , $\mathbf{V_P}$, and $\mathbf{V_F}$ to define local inertial reference frames.

I first calculated that V_G , $\mathbf{V_P}$, and $\mathbf{V_F}$ could define local inertial reference frames after reading Feynman[2] in about 1980 when I worked at the Stanford Linear Accelerator Center. I set the idea aside, until just recently, in order to work on artificial neural networks, computer languages, and 3D graphics.

6 Discussion

There has been much recent interest in experiments that claim to observe violations of Lorentz invariance in non-electromagnetic physics[1]. This new theory is compatible with those results because it retains the Lorentz invariance of Maxwell's equations without requiring the other laws of physics to be Lorentz invariant. It may even be that the weak and strong forces are effects of the momentum and force potentials.

There has also been much recent interest in decoupling space and time at high energies in theories of quantum gravity[3]. Since this new theory decouples space and time at all energies, it might provide an easier path to a theory of quantum gravity.

There is also a constant interest in unifying gravity with the rest of physics[4]. This new theory in a sense completes Maxwell's equations by defining inertial reference frames in terms of the gravitational, momentum, and force potentials, and so might be the basis for the Grand Unified Theory.

In the past the discovery of new potentials has explained previous parity violations. For example, Newton's discovery of the laws of gravitation explained an up-down parity violation: apples prefer to fall down. The discovery of the laws of magnetism explained a directional parity violation: compass needles prefer to point north.

The fact that we currently observe a left-right parity violation makes me think of the cross product of two vectors, and the momentum and force potentials provide two new vectors to cross. So perhaps the magnitude of left-right parity violation that we measure on Earth is an astronomical or cosmological phenomenon due to the momentum and force potentials generated by the motions of the Earth, Moon, Sun, or galaxies.

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