General Relativity in Absolute Space and Absolute Time

David B. Parker*
pgu.org
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A problem with accurately simulationing General Relativity is that it is computationally inefficient. The problem arises because accurate simulations necessitate nonuniform step sizes in both space and time. This paper shows how to solve that problem by implementing General Relativity in absolute space and absolute time. Using a uniform 3-dimensional grid with a uniform time step should greatly speed up gravitational simulations, especially in high density high force situations such as QCD nucleon modeling. The most important result in this letter is the 3-dimensional force equation for absolute gravity.

INTRODUCTION

Accurate simulations of high density high force situations are computationally expensive because of the non-uniform space and time steps that General Relativity necessitates. This letter show how to implement General Relativity in absolute space and absolute time so that General Relativity simulations can be accurately run on a uniform 3-dimensional grid with a uniform time step.

Here is the most important result in this letter, the force equation for absolute gravity:

$$\frac{\mathrm{d}^2 r^i}{\mathrm{d}t^2} = -\left(g^{i\sigma} - \frac{1}{c} g^{0\sigma} \frac{\mathrm{d}r^i}{\mathrm{d}t}\right) [\mu\nu, \sigma] \frac{\mathrm{d}r^\mu}{\mathrm{d}t} \frac{\mathrm{d}r^\nu}{\mathrm{d}t} \tag{1}$$

Anyone already familiar enough with General Relativity can probably skip directly to equation (9) in the derivation, since the intervening material is mainly review.

Equation (1) is written using Einstein's summation convention. The indexes μ, ν, and σ run from 0 to 3. The index i runs from 1 to 3. Equation (1) represents a system of three equations for $i = 1, 2, 3$. t is absolute time. The r^i are the absolute coordinates (r_x, r_y, r_z) of a particle in an absolute (x, y, z) coordinate system, so that $\frac{\mathrm{d}^2 r^i}{\mathrm{d}t^2}$ is the absolute acceleration of a particle that is moving at absolute velocity $\frac{\mathrm{d}r^i}{\mathrm{d}t}$. The $[\mu\nu, \sigma]$ are Christoffel symbols of the first kind. Inside the Christoffel symbols, it is important to remember that $x^0 = ct$ and $x^1, x^2, x^3 = x, y, z$. In the summations on the right hand side, it is important to remember that $r^0 = ct$, so that $\frac{\mathrm{d}r^0}{\mathrm{d}t} = c$. The units for t and x, y, z are ordinary time and length units; for example, seconds and meters. No constants have been set to 1. Mass does not appear in equation (1) because gravitational acceleration is independent of mass.

In absolute gravity, the field equation for the generation and propagation of $g_{\mu\nu}$ is the same as the Einstein equation in General Relativity. There is an important difference of interpretation, though. In absolute gravity, $g_{\mu\nu}$ is no longer a metric; it is a set of 10 potentials (taking advantage of the symmetry of $g_{\mu\nu}$). Computationally, now that the force equation is in terms of absolute space and absolute time, the Einstein equation can also be simulated on a uniform 3-dimensional grid with uniform time step.

I will be using equation (1) in my own simulations. I hope others find it as useful.

DERIVING THE ABSOLUTE GRAVITY FORCE EQUATION

We start from the geodesic equation:

$$\frac{\mathrm{d}^2 r^\alpha}{\mathrm{d}\tau^2} = -\Gamma^\alpha_{\mu\nu} \frac{\mathrm{d}r^\mu}{\mathrm{d}\tau} \frac{\mathrm{d}r^\nu}{\mathrm{d}\tau} \tag{2}$$

τ is the proper time as measured by a standard clock moving with the particle. The trajectory of the particle in four dimensions is $r^\alpha = (r^0, r^1, r^2, r^3)$ where (r^0, r^1, r^2, r^3) are the coordinates of the particle in the coordinate system $x^\alpha = (x^0, x^1, x^2, x^3)$. x^0 is taken to be time coordinate $x^0 = ct$, where in General Relativity x^0 and/or t are called the coordinate time. Later we will take $r^0 = x^0 = ct$, with t the absolute time, and (x^1, x^2, x^3) to be the absolute (x, y, z) coordinate system, but there is work to be done before we can make that leap.

The Christoffel symbol of the second kind in equation (2), $\Gamma^{\alpha}_{\mu\nu}$, conflates the role of the inverse metric, $g^{\alpha\beta}$ with the derivatives of the metric $g_{\alpha\beta}$ in calculating the geodesic:

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\sigma} \left(\frac{\partial g_{\sigma\mu}}{\partial x^{\nu}} + \frac{\partial g_{\nu\sigma}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}} \right) \tag{3}$$

We are going to revert to a somewhat more archaic notation (see [1], equations (20d) through (23)) using Christoffel symbols of the first kind, written as $[\mu\nu, \sigma]$, to separate the inverse metric from the derivatives of the metric:

$$[\mu\nu, \sigma] = \frac{1}{2} \left(\frac{\partial g_{\sigma\mu}}{\partial x^{\nu}} + \frac{\partial g_{\nu\sigma}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\sigma}} \right) \tag{4}$$

Substituting equation (4) into equation (3) for $\Gamma^{\alpha}_{\mu\nu}$ gives:

$$\Gamma^{\alpha}_{\mu\nu} = g^{\alpha\sigma} [\mu\nu, \sigma] \tag{5}$$

Substituting equation (5) into equation (2) for the geodesic gives:

$$\frac{\mathrm{d}^2 r^{\alpha}}{\mathrm{d}\tau^2} = -g^{\alpha\sigma} [\mu\nu, \sigma] \frac{\mathrm{d}r^{\mu}}{\mathrm{d}\tau} \frac{\mathrm{d}r^{\nu}}{\mathrm{d}\tau} \tag{6}$$

We now want to get rid of the derivatives with respect to τ (the proper time), and replace them with derivatives with respect to x^0 (the coordinate time). Applying the chain rule for derivatives gives:

$$\frac{\mathrm{d}r^{\alpha}}{\mathrm{d}\tau} = \frac{\mathrm{d}r^{\alpha}}{\mathrm{d}x^0} \frac{\mathrm{d}x^0}{\mathrm{d}\tau}, \qquad \frac{\mathrm{d}^2 r^{\alpha}}{\mathrm{d}\tau^2} = \left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau} \right)^2 \frac{\mathrm{d}^2 r^{\alpha}}{\mathrm{d}x^{0\,2}} + \frac{\mathrm{d}^2 x^0}{\mathrm{d}\tau^2} \frac{\mathrm{d}r^{\alpha}}{\mathrm{d}x^0} \tag{7}$$

Substituting the derivatives from equations (7) into equation (6):

$$\frac{\mathrm{d}^2 r^{\alpha}}{\mathrm{d}x^{0\,2}} \left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau} \right)^2 + \frac{\mathrm{d}^2 x^0}{\mathrm{d}\tau^2} \frac{\mathrm{d}r^{\alpha}}{\mathrm{d}x^0} = -g^{\alpha\sigma} [\mu\nu, \sigma] \frac{\mathrm{d}r^{\mu}}{\mathrm{d}x^0} \frac{\mathrm{d}r^{\nu}}{\mathrm{d}x^0} \left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau} \right)^2 \tag{8}$$

Multiplying both sides of equation (8) by $\left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau} \right)^{-2}$:

$$\frac{\mathrm{d}^2 r^{\alpha}}{\mathrm{d}x^{0\,2}} + \left\{ \frac{\mathrm{d}^2 x^0}{\mathrm{d}\tau^2} \left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau} \right)^{-2} \right\} \frac{\mathrm{d}r^{\alpha}}{\mathrm{d}x^0} = -g^{\alpha\sigma} [\mu\nu, \sigma] \frac{\mathrm{d}r^{\mu}}{\mathrm{d}x^0} \frac{\mathrm{d}r^{\nu}}{\mathrm{d}x^0} \tag{9}$$

We are entering the crucial steps that will separate space from time so that we can use a uniform time step. Equation (9) has a complicated factor in curly braces. Actually, equation (9) represents four equations, one for each of $\alpha = 0, 1, 2, 3$. The complicated factor, because it depends only on x^0 and τ, is the same for all four equations. We are going to sacrifice one of the equations to solve for the complicated factor. In particular, we are going to sacrifice the equation for $\alpha = 0$.

Separating equation (9) into separate equations for $\alpha = 0$ and $\alpha = 1, 2, 3$, and then using i instead of α to remind ourselves that the index now only goes from 1 to 3:

$$\frac{\mathrm{d}^2 r^{0}}{\mathrm{d}x^{0\,2}} + \left\{ \frac{\mathrm{d}^2 x^0}{\mathrm{d}\tau^2} \left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau} \right)^{-2} \right\} \frac{\mathrm{d}r^{0}}{\mathrm{d}x^0} = -g^{0\sigma} [\mu\nu, \sigma] \frac{\mathrm{d}r^{\mu}}{\mathrm{d}x^0} \frac{\mathrm{d}r^{\nu}}{\mathrm{d}x^0} \tag{10a}$$

$$\frac{\mathrm{d}^2 r^{i}}{\mathrm{d}x^{0\,2}} + \left\{ \frac{\mathrm{d}^2 x^0}{\mathrm{d}\tau^2} \left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau} \right)^{-2} \right\} \frac{\mathrm{d}r^{i}}{\mathrm{d}x^0} = -g^{i\sigma} [\mu\nu, \sigma] \frac{\mathrm{d}r^{\mu}}{\mathrm{d}x^0} \frac{\mathrm{d}r^{\nu}}{\mathrm{d}x^0} \tag{10b}$$

To solve equation (10a) for the complicated factor, we now choose time to be absolute. In particular, we choose that both the particle time r^0 and the coordinate time x^0 are to be measured in absolute time, hence:

$$\frac{\mathrm{d}r^0}{\mathrm{d}x^0} = 1, \qquad \frac{\mathrm{d}^2 r^0}{\mathrm{d}x^{0\,2}} = 0 \tag{11}$$

We don't necessarily need to choose space to be absolute for this derivation; however, we will need to choose space to be absolute when electromagnetism is added.

Solving for the complicated factor by substituting equations (11) into equation (10a) gives:

$$\left\{\frac{\mathrm{d}^2 x^0}{\mathrm{d}\tau^2}\left(\frac{\mathrm{d}x^0}{\mathrm{d}\tau}\right)^{-2}\right\} = -g^{0\sigma}[\mu\nu,\sigma]\frac{\mathrm{d}r^\mu}{\mathrm{d}x^0}\frac{\mathrm{d}r^\nu}{\mathrm{d}x^0} \tag{12}$$

Substituting equation (12) into equation (10b) and moving the term containing the complicated factor to the right hand side gives:

$$\frac{\mathrm{d}^2 r^i}{\mathrm{d}{x^0}^2} = -\left(g^{i\sigma} - g^{0\sigma}\frac{\mathrm{d}r^i}{\mathrm{d}x^0}\right)[\mu\nu,\sigma]\frac{\mathrm{d}r^\mu}{\mathrm{d}x^0}\frac{\mathrm{d}r^\nu}{\mathrm{d}x^0} \tag{13}$$

Finally, replacing x^0 with ct and multiplying both sides of equation (13) by c^2 gives equation (1).

* Electronic address: daveparker@pgu.org

[1] Einstein, A., "The Foundation of the General Theory of Relativity", 1916, pgs 111-164, translated by Perrett, W., and Jeffery G. B.,