How to derive the field equations for three new kinds gravitons

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This article shows how to derive the field equations for three new kinds of gravitons. Starting from the Einstein equation $G_{\alpha\beta} = (8\pi K/c^4)T_{\alpha\beta}$, an outline of the derivation is: first, find and prove that the highest-order independent pure (HIP) time derivatives of $g_{\mu\nu}$ that appear in the Einstein tensor $G_{\alpha\beta}$ are the 10 partial derivatives $\partial_0 g_{00}$, $\partial_0 g_{10}$, $\partial_0 g_{20}$, $\partial_0 g_{30}$, $\partial^2_{00} g_{11}$, $\partial^2_{00} g_{21}$, $\partial^2_{00} g_{31}$, $\partial^2_{00} g_{22}$, $\partial^2_{00} g_{32}$, and $\partial^2_{00} g_{33}$. Second, prove that the Einstein tensor is linear in those derivatives. Third, solve the Einstein equation for those derivatives. Finally, set the coordinate system to absolute space and time, which converts the partial derivatives into total derivatives, yielding the field equations for a scalar, a vector, and a matrix graviton. These new gravitons are important because they may be able to explain phenomena ranging from two-slit diffraction, to parity violation, to the Heisenberg uncertainty principle, to dark energy, to dark matter, and to the periodic table of the particles.

Introduction. This article shows how to derive the field equations for three new kinds of gravitons. These new gravitons are important because they may be able to explain phenomena ranging from two-slit diffraction, to parity violation, to the Heisenberg uncertainty principle, to dark energy, to dark matter, and to the periodic table of the particles[1].

The main result of this article is the set of ordinary differential field equations for the new scalar, vector, and matrix gravitons:

$$\frac{\mathrm{d}g}{\mathrm{d}x^0} = g_\mathrm{i}, \qquad \frac{\mathrm{d}\mathbf{w}}{\mathrm{d}x^0} = \mathbf{w}_\mathrm{i}, \qquad \frac{\mathrm{d}^2\mathbf{S}}{\mathrm{d}(x^0)^2} = \mathbf{S}_\mathrm{i}, \tag{1}$$

where $x^0 = ct$; g_i, $\mathbf{w}_\mathrm{i}$, and $\mathbf{S}_\mathrm{i}$ are the mass/interaction terms; and the fields g, $\mathbf{w}$, and $\mathbf{S}$ correspond to blocks of the symmetric metric tensor $g_{\mu\nu}$:

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & g_{10} & g_{20} & g_{30} \\ g_{10} & g_{11} & g_{21} & g_{31} \\ g_{20} & g_{21} & g_{22} & g_{32} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} g & w_1 & w_2 & w_3 \\ w_1 & S_{11} & S_{21} & S_{31} \\ w_2 & S_{21} & S_{22} & S_{32} \\ w_3 & S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} g & \mathbf{w}^\intercal \\ \mathbf{w} & \mathbf{S} \end{bmatrix}. \tag{2}$$

The equations for g and $\mathbf{w}$ go as d/dt, so they are diffusive or Schrödinger-like. The equation for $\mathbf{S}$ goes as d/dt^2, so it is wavelike or Klein-Gordon-like.

A Maxima file is available to check the derivation[2].

The derivation starts from the Einstein equation[3]:

$$G_{\alpha\beta} = (8\pi K/c^4)T_{\alpha\beta}. \tag{3}$$

Outline of the eight steps in the derivation.

- (Steps 1-2) Find and prove that the highest-order independent pure (HIP) time (i.e. x^0) derivatives of $g_{\mu\nu}$ that appear in the Einstein tensor $G_{\alpha\beta}$ are the 10 partial derivatives
$$\begin{matrix} \partial_0 g_{00}, & \partial_0 g_{10}, & \partial_0 g_{20}, & \partial_0 g_{30}, & \partial^2_{00} g_{11}, \\ \partial^2_{00} g_{21}, & \partial^2_{00} g_{31}, & \partial^2_{00} g_{22}, & \partial^2_{00} g_{32}, & \partial^2_{00} g_{33}. \end{matrix} \tag{4}$$
- (Step 3) Prove that the Einstein tensor is linear in those derivatives.
- (Steps 4-7) Solve the Einstein equation for those derivatives.
- (Step 8) Set the coordinate system to absolute space and time, which converts the partial derivatives into total derivatives, yielding the graviton field equations (1).

Step 1: separate 1st and 2nd derivatives. To prove later theorems by inspection instead of by index gymnastics, transform the Einstein tensor $G_{\alpha\beta}$ to remove the derivatives of the inverse metric $g^{\mu\nu}$, and to separate the 1st and 2nd derivatives of $g_{\mu\nu}$. The Einstein tensor in terms of the Ricci tensor $R_{\alpha\beta}$ and Ricci scalar R is

$$G_{\alpha\beta} = R_{\alpha\beta} - \tfrac{1}{2}\, g_{\alpha\beta} R. \tag{5}$$

Expand the Ricci scalar $R = g^{\mu\nu} R_{\mu\nu}$:

$$G_{\alpha\beta} = R_{\alpha\beta} - \tfrac{1}{2}\, g_{\alpha\beta} g^{\mu\nu} R_{\mu\nu}. \tag{6}$$

The Ricci tensor in terms of the Christoffel symbol of the 2nd kind $\Gamma^\gamma_{\alpha\beta}$ is

$$R_{\alpha\beta} = \partial_\gamma \Gamma^\gamma_{\alpha\beta} - \partial_\beta \Gamma^\gamma_{\alpha\gamma} + \Gamma^\eta_{\alpha\beta}\Gamma^\gamma_{\gamma\eta} - \Gamma^\eta_{\alpha\gamma}\Gamma^\gamma_{\beta\eta}. \tag{7}$$

$\Gamma^\gamma_{\alpha\beta}$ in terms of the inverse metric $g^{\mu\nu}$ and the Christoffel symbol of the 1st kind $[\alpha\beta,\delta]$, and the derivative of $\Gamma^\gamma_{\alpha\beta}$ with respect to x^σ, are

$$\Gamma^\gamma_{\alpha\beta} = g^{\gamma\delta}[\alpha\beta,\delta], \tag{8}$$

$$\partial_\sigma \Gamma^\gamma_{\alpha\beta} = g^{\gamma\delta}\partial_\sigma [\alpha\beta,\delta] + \left(\partial_\sigma g^{\gamma\delta}\right)[\alpha\beta,\delta]. \tag{9}$$

$[\alpha\beta,\delta]$ and its derivative with respect to x^σ are

$$[\alpha\beta,\delta] = \tfrac{1}{2}\left(\partial_\beta g_{\alpha\delta} + \partial_\alpha g_{\beta\delta} - \partial_\delta g_{\alpha\beta}\right), \tag{10}$$

$$\partial_\sigma [\alpha\beta,\delta] = \tfrac{1}{2}\left(\partial^2_{\sigma\beta} g_{\alpha\delta} + \partial^2_{\sigma\alpha} g_{\beta\delta} - \partial^2_{\sigma\delta} g_{\alpha\beta}\right). \tag{11}$$

To make it easier to simplify (15) to (17), change the dummy index in the second term of (9) from δ to ϵ:

$$\partial_\sigma \Gamma^\gamma_{\alpha\beta} = g^{\gamma\delta}\partial_\sigma [\alpha\beta,\delta] + \left(\partial_\sigma g^{\gamma\epsilon}\right)[\alpha\beta,\epsilon]. \tag{12}$$

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The derivative of the inverse metric $\partial_\sigma g^{\gamma\epsilon}$ in terms of derivatives of the metric is[4]

$$\partial_\sigma g^{\gamma\epsilon} = -g^{\gamma\delta} g^{\epsilon\eta} \partial_\sigma g_{\delta\eta}. \tag{13}$$

Plug (13) into (12) to eliminate the derivatives of the inverse metric:

$$\partial_\sigma \Gamma^\gamma_{\alpha\beta} = g^{\gamma\delta} \partial_\sigma [\alpha\beta,\delta] - g^{\gamma\delta} g^{\epsilon\eta} \left(\partial_\sigma g_{\delta\eta}\right) [\alpha\beta,\epsilon]. \tag{14}$$

Plug (14) and (8) into (7), picking the dummy indices judiciously:

$$\begin{aligned} R_{\alpha\beta} = &\left(g^{\gamma\delta}\partial_\gamma [\alpha\beta,\delta] - g^{\gamma\delta} g^{\epsilon\eta} \left(\partial_\gamma g_{\delta\eta}\right)[\alpha\beta,\epsilon]\right) \\ &- \left(g^{\gamma\delta}\partial_\beta [\alpha\gamma,\delta] - g^{\gamma\delta} g^{\epsilon\eta} \left(\partial_\beta g_{\delta\eta}\right)[\alpha\gamma,\epsilon]\right) \\ &+ g^{\epsilon\eta}[\alpha\beta,\epsilon] g^{\gamma\delta}[\gamma\eta,\delta] - g^{\epsilon\eta}[\alpha\gamma,\epsilon] g^{\gamma\delta}[\beta\eta,\delta] \qquad (15) \\ = &\, g^{\gamma\delta}\left(\partial_\gamma [\alpha\beta,\delta] - \partial_\beta [\alpha\gamma,\delta]\right) \\ &+ g^{\gamma\delta} g^{\epsilon\eta}\big([\alpha\gamma,\epsilon]\left(-[\beta\eta,\delta] + \partial_\beta g_{\delta\eta}\right) \\ &\qquad - [\alpha\beta,\epsilon]\left(-[\gamma\eta,\delta] + \partial_\gamma g_{\delta\eta}\right)\big) \qquad (16) \\ = &\, g^{\gamma\delta}\left(\partial_\gamma [\alpha\beta,\delta] - \partial_\beta [\alpha\gamma,\delta]\right) \\ &+ g^{\gamma\delta} g^{\epsilon\eta}\left([\alpha\gamma,\epsilon][\beta\delta,\eta] - [\alpha\beta,\epsilon][\gamma\delta,\eta]\right). \qquad (17) \end{aligned}$$

Let:

$$Q_{\alpha\beta(\gamma\delta)} = g^{\gamma\delta}\left(\partial_\gamma [\alpha\beta,\delta] - \partial_\beta [\alpha\gamma,\delta]\right), \tag{18}$$

$$P_{\alpha\beta(\gamma\delta\epsilon\eta)} = g^{\gamma\delta} g^{\epsilon\eta}\left([\alpha\gamma,\epsilon][\beta\delta,\eta] - [\alpha\beta,\epsilon][\gamma\delta,\eta]\right). \tag{19}$$

In the subscripts of $Q_{\alpha\beta(\gamma\delta)}$ and $P_{\alpha\beta(\gamma\delta\epsilon\eta)}$, the dummy indices are inside the parentheses. The free indices preceed the parenthesis. $Q_{\alpha\beta(\gamma\delta)}$ contains only 2^{nd} derivatives, and $P_{\alpha\beta(\gamma\delta\epsilon\eta)}$ contains only 1^{st} derivatives. Plug (18) and (19) into (17) to explicitly separate the 1^{st} and 2^{nd} derivatives in $R_{\alpha\beta}$:

$$R_{\alpha\beta} = Q_{\alpha\beta(\gamma\delta)} + P_{\alpha\beta(\gamma\delta\epsilon\eta)}. \tag{20}$$

Plug (20) into (6) to explicitly separate the 1^{st} and 2^{nd} derivatives in $G_{\alpha\beta}$:

$$\begin{aligned} G_{\alpha\beta} = &\, Q_{\alpha\beta(\gamma\delta)} - \tfrac{1}{2} g_{\alpha\beta} g^{\mu\nu} Q_{\mu\nu(\gamma\delta)} \\ &+ P_{\alpha\beta(\gamma\delta\epsilon\eta)} - \tfrac{1}{2} g_{\alpha\beta} g^{\mu\nu} P_{\mu\nu(\gamma\delta\epsilon\eta)}. \end{aligned} \tag{21}$$

Step 2: find and prove HIP time derivatives. To find the HIP time derivatives of $g_{\mu\nu}$ in $G_{\alpha\beta}$ (4) using (21), divide each of the ranges $0 \le \alpha, \beta, \gamma, \delta, \epsilon, \eta \le 3$ into two subranges: 0 and $1 \le i, j, k, l, m, n \le 3$. Dividing each range into two subranges expands $Q_{\alpha\beta(\gamma\delta)}$ into $2^4 = 16$ terms, and expands $P_{\alpha\beta(\gamma\delta\epsilon\eta)}$ into $2^6 = 64$ terms. The expanded terms of $Q_{\alpha\beta(\gamma\delta)}$ and $P_{\alpha\beta(\gamma\delta\epsilon\eta)}$ are tabulated in Appendix I and Appendix II.

Theorem 1: The HIP time derivatives of $g_{\mu\nu}$ in $G_{\alpha\beta}$ are (4).

Proof: By inspection, $Q_{00(kl)}$ (for example) contains the 2^{nd} derivatives $\partial^2_{00} g_{kl} = {}^{kl}_{00}$, but no term of $Q_{\alpha\beta(\gamma\delta)}$ contains any of the 2^{nd} derivatives $\partial^2_{00} g_{\mu 0} = {}^{\mu 0}_{00}$. Therefore, the HIP time derivatives of $g_{\mu 0}$ are $\partial_0 g_{\mu 0} = {}^{\mu 0}_{0}$ because, for example, $P_{00(kl00)}$ contains $\partial_0 g_{00} = {}^{00}_{0}$ and $P_{00(klm0)}$ contains $\partial_0 g_{m0} = {}^{m0}_{0}$. QED

Step 3: prove linearity.

Theorem 2: $G_{\alpha\beta}$ is linear in the HIP time derivatives (4).

Proof: By inspection, $G_{\alpha\beta}$ (21) is linear in $Q_{\alpha\beta(\gamma\delta)}$, and all of the terms of $Q_{\alpha\beta(\gamma\delta)}$ are linear in all of the 2^{nd} derivatives, so $G_{\alpha\beta}$ is linear in all of the 2^{nd} derivatives in (4). Also by inspection, $G_{\alpha\beta}$ (21) is linear in $P_{\alpha\beta(\gamma\delta\epsilon\eta)}$, and in all of the terms of $P_{\alpha\beta(\gamma\delta\epsilon\eta)}$ there is no case where a derivative $\partial_0 g_{\mu 0} = {}^{\mu 0}_{0}$ multiplies a derivative $\partial_0 g_{\nu 0} = {}^{\nu 0}_{0}$ for $\mu, \nu = 0, i, j, k, l, m, n$, so $G_{\alpha\beta}$ is linear in all of the 1^{st} derivatives in (4). QED

Step 4: compress notation. Compress notation by subsuming the constant in the Einstein equation (3) into a scaled EMS tensor $U_{\alpha\beta}$:

$$U_{\alpha\beta} = (8\pi K/c^4) T_{\alpha\beta}. \tag{22}$$

The initial Einstein equation (3) becomes the scaled Einstein equation:

$$G_{\alpha\beta} = U_{\alpha\beta}. \tag{23}$$

Step 5: pack into 10-vectors. Pack the HIP time derivatives (4), and the corresponding elements of $G_{\alpha\beta}$ (21) and $U_{\alpha\beta}$ (22), into 10-vectors $\mathbf{d}$, $\mathbf{g}$, and $\mathbf{u}$:

$$\mathbf{d} = \begin{bmatrix} \partial_0 g_{00} & \partial_0 g_{10} & \partial_0 g_{20} & \partial_0 g_{30} & \partial^2_{00} g_{11} \\ \partial^2_{00} g_{21} & \partial^2_{00} g_{31} & \partial^2_{00} g_{22} & \partial^2_{00} g_{32} & \partial^2_{00} g_{33} \end{bmatrix}^\intercal, \tag{24}$$

$$\mathbf{g} = \begin{bmatrix} G_{00} & G_{10} & G_{20} & G_{30} & G_{11} \\ G_{21} & G_{31} & G_{22} & G_{32} & G_{33} \end{bmatrix}^\intercal, \tag{25}$$

$$\mathbf{u} = \begin{bmatrix} U_{00} & U_{10} & U_{20} & U_{30} & U_{11} \\ U_{21} & U_{31} & U_{22} & U_{32} & U_{33} \end{bmatrix}^\intercal. \tag{26}$$

Step 6: solve the Einstein equation. In 10-vector form, the scaled Einstein equation (23) becomes the vector Einstein equation:

$$\mathbf{g} = \mathbf{u}. \tag{27}$$

From Step 3, $\mathbf{g}$ is linear in $\mathbf{d}$. $\mathbf{u}$ will be discussed in more detail in other work to appear, but to maximize the generality of this derivation, allow that $\mathbf{u}$ is also linear in $\mathbf{d}$ and has the same HIP time derivatives. From the linearity of $\mathbf{g}$ and $\mathbf{u}$ in $\mathbf{d}$, we can write $\mathbf{g}$ and $\mathbf{u}$ as

$$\mathbf{g} = \mathbf{M}\mathbf{d} + \mathbf{m}, \tag{28}$$

$$\mathbf{u} = \mathbf{N}\mathbf{d} + \mathbf{n}, \tag{29}$$

where $\mathbf{M}$ and $\mathbf{N}$ are 10×10 matrices, and $\mathbf{m}$ and $\mathbf{n}$ are 10-vectors. Plug (28) and (29) into (27):

$$\mathbf{M}\mathbf{d} + \mathbf{m} = \mathbf{N}\mathbf{d} + \mathbf{n}. \tag{30}$$

Solve for $\mathbf{d}$:

$$\mathbf{d} = (\mathbf{M} - \mathbf{N})^{-1} (\mathbf{n} - \mathbf{m}). \tag{31}$$

Let $\mathbf{i}$ be the 10-vector of mass/interaction terms from (31):

$$\mathbf{i} = (\mathbf{M} - \mathbf{N})^{-1} (\mathbf{n} - \mathbf{m}). \tag{32}$$

The solved Einstein equation becomes

$$\mathbf{d} = \mathbf{i}. \tag{33}$$

Step 7: unpack 10-vectors. Unpack the 10-vectors $\mathbf{d}$ (24) and $\mathbf{i}$ (32) into symmetric 4×4 matrices $\mathbf{D}$ and $\mathbf{I}$, then rename the elements to put them into block matrix form:

$$\begin{aligned}\mathbf{D} &= \begin{bmatrix} \partial_0 g_{00} & \partial_0 g_{10} & \partial_0 g_{20} & \partial_0 g_{30} \\ \partial_0 g_{10} & \partial_{00}^2 g_{11} & \partial_{00}^2 g_{21} & \partial_{00}^2 g_{31} \\ \partial_0 g_{20} & \partial_{00}^2 g_{21} & \partial_{00}^2 g_{22} & \partial_{00}^2 g_{32} \\ \partial_0 g_{30} & \partial_{00}^2 g_{31} & \partial_{00}^2 g_{32} & \partial_{00}^2 g_{33} \end{bmatrix} \\ &= \begin{bmatrix} \partial_0 g & \partial_0 w_1 & \partial_0 w_2 & \partial_0 w_3 \\ \partial_0 w_1 & \partial_{00}^2 S_{11} & \partial_{00}^2 S_{21} & \partial_{00}^2 S_{31} \\ \partial_0 w_2 & \partial_{00}^2 S_{21} & \partial_{00}^2 S_{22} & \partial_{00}^2 S_{32} \\ \partial_0 w_3 & \partial_{00}^2 S_{31} & \partial_{00}^2 S_{32} & \partial_{00}^2 S_{33} \end{bmatrix} \\ &= \begin{bmatrix} \partial_0 g & \partial_0 \mathbf{w}^\intercal \\ \partial_0 \mathbf{w} & \partial_{00}^2 \mathbf{S} \end{bmatrix},\end{aligned} \tag{34}$$

$$\begin{aligned}\mathbf{I} &= \begin{bmatrix} \mathbf{i}_0 & \mathbf{i}_1 & \mathbf{i}_2 & \mathbf{i}_3 \\ \mathbf{i}_1 & \mathbf{i}_4 & \mathbf{i}_5 & \mathbf{i}_6 \\ \mathbf{i}_2 & \mathbf{i}_5 & \mathbf{i}_7 & \mathbf{i}_8 \\ \mathbf{i}_3 & \mathbf{i}_6 & \mathbf{i}_8 & \mathbf{i}_9 \end{bmatrix} = \begin{bmatrix} g_\mathrm{i} & w_{1_\mathrm{i}} & w_{2_\mathrm{i}} & w_{3_\mathrm{i}} \\ w_{1_\mathrm{i}} & S_{11_\mathrm{i}} & S_{21_\mathrm{i}} & S_{31_\mathrm{i}} \\ w_{2_\mathrm{i}} & S_{21_\mathrm{i}} & S_{22_\mathrm{i}} & S_{32_\mathrm{i}} \\ w_{3_\mathrm{i}} & S_{31_\mathrm{i}} & S_{32_\mathrm{i}} & S_{33_\mathrm{i}} \end{bmatrix} \\ &= \begin{bmatrix} g_\mathrm{i} & \mathbf{w}_\mathrm{i}^\intercal \\ \mathbf{w}_\mathrm{i} & \mathbf{S}_\mathrm{i} \end{bmatrix}.\end{aligned} \tag{35}$$

The solved Einstein equation (33) becomes

$$\mathbf{D} = \mathbf{I}. \tag{36}$$

Equate corresponding blocks in $\mathbf{D}$ (34) and $\mathbf{I}$ (35) to get the unpacked Einstein equations:

$$\partial_0 g = g_\mathrm{i}, \quad \partial_0 \mathbf{w} = \mathbf{w}_\mathrm{i}, \quad \partial_{00}^2 \mathbf{S} = \mathbf{S}_\mathrm{i}. \tag{37}$$

The unpacked Einstein equations (37) are mathematically the same as the initial Einstein equation (3), but in a different form.

Step 8: absolute space and time. Convert the partial derivatives in (37) (and the partial derivatives implicit in g_i, $\mathbf{w}_\mathrm{i}$, and $\mathbf{S}_\mathrm{i}$) into total derivatives. To do that, set the coordinate system to absolute space and time:

$$x^\delta = [ct, x, y, z]^\intercal. \tag{38}$$

In absolute space and time the coordinates do not vary with respect to each other:

$$\frac{\mathrm{d}x^\gamma}{\mathrm{d}x^\delta} = \begin{cases} 1, & \text{if } \gamma = \delta, \\ 0, & \text{if } \gamma \neq \delta. \end{cases} \tag{39}$$

1st partial derivatives become 1st total derivatives. To see this, use the formula for total derivatives:

$$\begin{aligned}\frac{\mathrm{d}g_{\mu\nu}}{\mathrm{d}x^\alpha} &= \frac{\partial g_{\mu\nu}}{\partial x^0}\frac{\mathrm{d}x^0}{\mathrm{d}x^\alpha} + \frac{\partial g_{\mu\nu}}{\partial x^1}\frac{\mathrm{d}x^1}{\mathrm{d}x^\alpha} + \frac{\partial g_{\mu\nu}}{\partial x^2}\frac{\mathrm{d}x^2}{\mathrm{d}x^\alpha} + \frac{\partial g_{\mu\nu}}{\partial x^3}\frac{\mathrm{d}x^3}{\mathrm{d}x^\alpha} \\ &= \frac{\partial g_{\mu\nu}}{\partial x^\alpha}.\end{aligned} \tag{40}$$

Similarly for 2nd derivatives:

$$\begin{aligned}\frac{\mathrm{d}^2 g_{\mu\nu}}{\mathrm{d}x^\beta\,\mathrm{d}x^\alpha} &= \frac{\mathrm{d}}{\mathrm{d}x^\beta}\frac{\mathrm{d}g_{\mu\nu}}{\mathrm{d}x^\alpha} = \frac{\mathrm{d}}{\mathrm{d}x^\beta}\frac{\partial g_{\mu\nu}}{\partial x^\alpha} \\ &= \frac{\partial^2 g_{\mu\nu}}{\partial x^0 \partial x^\alpha}\frac{\mathrm{d}x^0}{\mathrm{d}x^\beta} + \frac{\partial^2 g_{\mu\nu}}{\partial x^1 \partial x^\alpha}\frac{\mathrm{d}x^1}{\mathrm{d}x^\beta} \\ &\quad + \frac{\partial^2 g_{\mu\nu}}{\partial x^2 \partial x^\alpha}\frac{\mathrm{d}x^2}{\mathrm{d}x^\beta} + \frac{\partial^2 g_{\mu\nu}}{\partial x^3 \partial x^\alpha}\frac{\mathrm{d}x^3}{\mathrm{d}x^\beta} \\ &= \frac{\partial^2 g_{\mu\nu}}{\partial x^\beta \partial x^\alpha}.\end{aligned} \tag{41}$$

In absolute space and time, the unpacked Einstein equations (37) become the ordinary differential field equations for the three new gravitons (1):

$$\frac{\mathrm{d}g}{\mathrm{d}x^0} = g_\mathrm{i}, \qquad \frac{\mathrm{d}\mathbf{w}}{\mathrm{d}x^0} = \mathbf{w}_\mathrm{i}, \qquad \frac{\mathrm{d}^2\mathbf{S}}{\mathrm{d}(x^0)^2} = \mathbf{S}_\mathrm{i}. \tag{42}$$

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[1] Parker, D.B., "Parity violation is evidence that our universe is inside an extremal Kerr black hole (plus QEG)", poster, APS April Meeting, 2024, `https://pgu.org`

[2] Parker, D.B., "How to derive the field equations for three new kinds of gravitons", supplemental Maxima file, Dec 2024, `https://pgu.org`

[3] Hartle, J.B., *Gravity: An Introduction to Einstein's General Relativity*, 2021, pg 483, eq (22.51) ff

[4] Einstein, A., "The Foundation of the General Theory of Relativity", in *The Principle of Relativity*, Dover Publications, 1952, pg 137, eq (31)

Appendix I: terms in the expansion of $Q_{\alpha\beta(\gamma\delta)}$

Compressed notation:

$$ {}^{\alpha\beta}_{\gamma\delta} = \begin{cases} \partial^2_{\gamma\delta} g_{\alpha\beta} = \frac{\partial^2 g_{\alpha\beta}}{\partial x_\gamma \partial x_\delta}, & \text{for general relativity,} \\ \mathrm{d}^2_{\gamma\delta} g_{\alpha\beta} = \frac{\mathrm{d}^2 g_{\alpha\beta}}{\mathrm{d}x_\gamma\, \mathrm{d}x_\delta}, & \text{for QEG.} \end{cases} \tag{43} $$

$$ Q_{00(00)} = Q_{00(0l)} = Q_{00(k0)} = 0, \tag{44} $$
$$ Q_{00(kl)} = \tfrac{1}{2}\, g^{kl}({}^{l0}_{0k} - {}^{00}_{kl} - {}^{kl}_{00} + {}^{k0}_{0l}), \tag{45} $$
$$ Q_{0j(00)} = 0, \tag{46} $$
$$ Q_{0j(0l)} = \tfrac{1}{2}\, g^{l0}({}^{jl}_{00} - {}^{j0}_{0l} - {}^{l0}_{0j} + {}^{00}_{jl}), \tag{47} $$
$$ Q_{0j(k0)} = 0, \tag{48} $$
$$ Q_{0j(kl)} = \tfrac{1}{2}\, g^{kl}({}^{jl}_{0k} - {}^{j0}_{kl} - {}^{kl}_{0j} + {}^{k0}_{jl}), \tag{49} $$
$$ Q_{i0(00)} = Q_{i0(0l)} = 0, \tag{50} $$
$$ Q_{i0(k0)} = \tfrac{1}{2}\, g^{k0}({}^{00}_{ik} - {}^{i0}_{0k} - {}^{k0}_{0i} + {}^{ik}_{00}), \tag{51} $$
$$ Q_{i0(kl)} = \tfrac{1}{2}\, g^{kl}({}^{l0}_{ik} - {}^{i0}_{kl} - {}^{kl}_{0i} + {}^{ik}_{0l}), \tag{52} $$
$$ Q_{ij(00)} = \tfrac{1}{2}\, g^{00}({}^{j0}_{0i} - {}^{ij}_{00} - {}^{00}_{ij} + {}^{i0}_{0j}), \tag{53} $$
$$ Q_{ij(0l)} = \tfrac{1}{2}\, g^{l0}({}^{jl}_{0i} - {}^{ij}_{0l} - {}^{l0}_{ij} + {}^{i0}_{jl}), \tag{54} $$
$$ Q_{ij(k0)} = \tfrac{1}{2}\, g^{k0}({}^{j0}_{ik} - {}^{ij}_{0k} - {}^{k0}_{ij} + {}^{ik}_{0j}), \tag{55} $$
$$ Q_{ij(kl)} = \tfrac{1}{2}\, g^{kl}({}^{jl}_{ik} - {}^{ij}_{kl} - {}^{kl}_{ij} + {}^{ik}_{jl}). \tag{56} $$

Appendix II: terms in the expansion of $P_{\alpha\beta(\gamma\delta\epsilon\eta)}$

Compressed notation:

$$ {}^{\alpha\beta}_{\gamma} = \begin{cases} \partial_\gamma g_{\alpha\beta} = \frac{\partial g_{\alpha\beta}}{\partial x_\gamma}, & \text{for general relativity,} \\ \mathrm{d}_\gamma g_{\alpha\beta} = \frac{\mathrm{d} g_{\alpha\beta}}{\mathrm{d}x_\gamma}. & \text{for QEG.} \end{cases} \tag{57} $$

$$ P_{00(0000)}=P_{00(000n)}=P_{00(00m0)}=0, \tag{58} $$
$$ P_{00(00mn)}=P_{00(0l00)}=P_{00(0l0n)}=0, \tag{59} $$
$$ P_{00(0lm0)}=P_{00(0lmn)}=P_{00(k000)}=0, \tag{60} $$
$$ P_{00(k00n)}=\tfrac14\, g^{k0}g^{n0}({}^{00}_{k}(2\,{}^{n0}_{0}-{}^{00}_{n})-{}^{00}_{0}({}^{kn}_{0}+{}^{n0}_{k}-{}^{k0}_{n})), \tag{61} $$
$$ P_{00(k0m0)}=\tfrac14\, g^{k0}g^{m0}(({}^{m0}_{k}+{}^{km}_{0}-{}^{k0}_{m}){}^{00}_{0}-(2\,{}^{m0}_{0}-{}^{00}_{m}){}^{00}_{k}), \tag{62} $$
$$ P_{00(k0mn)}=0, \tag{63} $$
$$ P_{00(kl00)}=\tfrac14\, g^{kl}g^{00}({}^{00}_{k}{}^{00}_{l}-{}^{00}_{0}({}^{k0}_{l}+{}^{l0}_{k}-{}^{kl}_{0})), \tag{64} $$
$$ P_{00(kl0n)}=\tfrac14\, g^{kl}g^{n0}({}^{00}_{k}({}^{n0}_{l}+{}^{ln}_{0}-{}^{l0}_{n})-{}^{00}_{0}({}^{kn}_{l}+{}^{ln}_{k}-{}^{kl}_{n})), \tag{65} $$
$$ P_{00(klm0)}=\tfrac14\, g^{kl}g^{m0}(({}^{m0}_{k}+{}^{km}_{0}-{}^{k0}_{m}){}^{00}_{l} -(2\,{}^{m0}_{0}-{}^{00}_{m})({}^{k0}_{l}+{}^{l0}_{k}-{}^{kl}_{0})), \tag{66} $$
$$ P_{00(klmn)}=\tfrac14\, g^{kl}g^{mn}(({}^{m0}_{k}+{}^{km}_{0}-{}^{k0}_{m})({}^{n0}_{l}+{}^{ln}_{0}-{}^{l0}_{n}) -({}^{kn}_{l}+{}^{ln}_{k}-{}^{kl}_{n})(2\,{}^{m0}_{0}-{}^{00}_{m})), \tag{67} $$
$$ P_{0j(0000)}=0, \tag{68} $$
$$ P_{0j(000n)}=\tfrac14\, g^{00}g^{n0}({}^{00}_{0}({}^{jn}_{0}+{}^{n0}_{j}-{}^{j0}_{n})-{}^{00}_{j}(2\,{}^{n0}_{0}-{}^{00}_{n})), \tag{69} $$
$$ P_{0j(00m0)}=\tfrac14\, g^{00}g^{m0}((2\,{}^{m0}_{0}-{}^{00}_{m}){}^{00}_{j}-({}^{m0}_{j}+{}^{jm}_{0}-{}^{j0}_{m}){}^{00}_{0}), \tag{70} $$
$$ P_{0j(00mn)}=0, \tag{71} $$
$$ P_{0j(0l00)}=\tfrac14\, g^{l0}g^{00}({}^{00}_{0}({}^{j0}_{l}+{}^{l0}_{j}-{}^{jl}_{0})-{}^{00}_{j}{}^{00}_{l}), \tag{72} $$
$$ P_{0j(0l0n)}=\tfrac14\, g^{l0}g^{n0}({}^{00}_{0}({}^{jn}_{l}+{}^{ln}_{j}-{}^{jl}_{n})-{}^{00}_{j}({}^{n0}_{l}+{}^{ln}_{0}-{}^{l0}_{n})), \tag{73} $$
$$ P_{0j(0lm0)}=\tfrac14\, g^{l0}g^{m0}((2\,{}^{m0}_{0}-{}^{00}_{m})({}^{j0}_{l}+{}^{l0}_{j}-{}^{jl}_{0}) -({}^{m0}_{j}+{}^{jm}_{0}-{}^{0j}_{m}){}^{00}_{l}), \tag{74} $$
$$ P_{0j(0lmn)}=\tfrac14\, g^{l0}g^{mn}((2\,{}^{m0}_{0}-{}^{00}_{m})({}^{jn}_{l}+{}^{ln}_{j}-{}^{jl}_{n}) -({}^{m0}_{j}+{}^{jm}_{0}-{}^{j0}_{m})({}^{n0}_{l}+{}^{ln}_{0}-{}^{l0}_{n})), \tag{75} $$
$$ P_{0j(k000)}=0, \tag{76} $$
$$ P_{0j(k00n)}=\tfrac14\, g^{k0}g^{n0}({}^{00}_{k}({}^{jn}_{0}+{}^{n0}_{j}-{}^{j0}_{n})-{}^{00}_{j}({}^{kn}_{0}+{}^{n0}_{k}-{}^{k0}_{n})), \tag{77} $$
$$ P_{0j(k0m0)}=\tfrac14\, g^{k0}g^{m0}(({}^{m0}_{k}+{}^{km}_{0}-{}^{k0}_{m}){}^{00}_{j} -({}^{m0}_{j}+{}^{jm}_{0}-{}^{j0}_{m}){}^{00}_{k}), \tag{78} $$
$$ P_{0j(k0mn)}=0, \tag{79} $$
$$ P_{0j(kl00)}=\tfrac14\, g^{kl}g^{00}({}^{00}_{k}({}^{j0}_{l}+{}^{l0}_{j}-{}^{jl}_{0})-{}^{00}_{j}({}^{k0}_{l}+{}^{l0}_{k}-{}^{kl}_{0})), \tag{80} $$
$$ P_{0j(kl0n)}=\tfrac14\, g^{kl}g^{n0}({}^{00}_{k}({}^{jn}_{l}+{}^{ln}_{j}-{}^{jl}_{n})-{}^{00}_{j}({}^{kn}_{l}+{}^{ln}_{k}-{}^{kl}_{n})), \tag{81} $$
$$ P_{0j(klm0)}=\tfrac14\, g^{kl}g^{m0}(({}^{m0}_{k}+{}^{km}_{0}-{}^{k0}_{m})({}^{j0}_{l}+{}^{l0}_{j}-{}^{jl}_{0}) -({}^{m0}_{j}+{}^{jm}_{0}-{}^{j0}_{m})({}^{k0}_{l}+{}^{l0}_{k}-{}^{kl}_{0})), \tag{82} $$
$$ P_{0j(klmn)}=\tfrac14\, g^{kl}g^{mn}(({}^{m0}_{k}+{}^{km}_{0}-{}^{k0}_{m})({}^{jn}_{l}+{}^{ln}_{j}-{}^{jl}_{n}) -({}^{m0}_{j}+{}^{jm}_{0}-{}^{j0}_{m})({}^{kn}_{l}+{}^{ln}_{k}-{}^{kl}_{n})), \tag{83} $$
$$ P_{i0(0000)}=P_{i0(000n)}=P_{i0(00m0)}=P_{i0(00mn)}=0, \tag{84} $$
$$ P_{i0(0l00)}=P_{i0(0l0n)}=P_{i0(0lm0)}=P_{i0(0lmn)}=0, \tag{85} $$
$$ P_{i0(k000)}=\tfrac14\, g^{k0}g^{00}(({}^{i0}_{k}+{}^{k0}_{i}-{}^{ik}_{0}){}^{00}_{0}-{}^{00}_{i}{}^{00}_{k}), \tag{86} $$
$$ P_{i0(k00n)}=\tfrac14\, g^{k0}g^{n0}(({}^{i0}_{k}+{}^{k0}_{i}-{}^{ik}_{0})(2\,{}^{n0}_{0}-{}^{00}_{n}) -{}^{00}_{i}({}^{kn}_{0}+{}^{n0}_{k}-{}^{k0}_{n})), \tag{87} $$
$$ P_{i0(k0m0)}=\tfrac14\, g^{k0}g^{m0}(({}^{im}_{k}+{}^{km}_{i}-{}^{ik}_{m}){}^{00}_{0} -({}^{im}_{0}+{}^{m0}_{i}-{}^{i0}_{m}){}^{00}_{k}), \tag{88} $$
$$ P_{i0(k0mn)}=\tfrac14\, g^{k0}g^{mn}(({}^{im}_{k}+{}^{km}_{i}-{}^{ik}_{m})(2\,{}^{n0}_{0}-{}^{00}_{n}) -({}^{im}_{0}+{}^{m0}_{i}-{}^{i0}_{m})({}^{kn}_{0}+{}^{n0}_{k}-{}^{k0}_{n})), \tag{89} $$
$$ P_{i0(kl00)}=\tfrac14\, g^{kl}g^{00}(({}^{i0}_{k}+{}^{k0}_{i}-{}^{ik}_{0}){}^{00}_{l}-{}^{00}_{i}({}^{k0}_{l}+{}^{l0}_{k}-{}^{kl}_{0})), \tag{90} $$
$$ P_{i0(kl0n)}=\tfrac14\, g^{kl}g^{n0}(({}^{i0}_{k}+{}^{k0}_{i}-{}^{ik}_{0})({}^{n0}_{l}+{}^{ln}_{0}-{}^{l0}_{n}) -{}^{00}_{i}({}^{kn}_{l}+{}^{ln}_{k}-{}^{kl}_{n})), \tag{91} $$
$$ P_{i0(klm0)}=\tfrac14\, g^{kl}g^{m0}(({}^{im}_{k}+{}^{km}_{i}-{}^{ik}_{m}){}^{00}_{l} -({}^{im}_{0}+{}^{m0}_{i}-{}^{i0}_{m})({}^{k0}_{l}+{}^{l0}_{k}-{}^{kl}_{0})), \tag{92} $$
$$ P_{i0(klmn)}=\tfrac14\, g^{kl}g^{mn}(({}^{im}_{k}+{}^{km}_{i}-{}^{ik}_{m})({}^{n0}_{l}+{}^{ln}_{0}-{}^{l0}_{n}) -({}^{im}_{0}+{}^{m0}_{i}-{}^{i0}_{m})({}^{kn}_{l}+{}^{ln}_{k}-{}^{kl}_{n})), \tag{93} $$
$$ P_{ij(0000)}=\tfrac14\, (g^{00})^2({}^{00}_{i}{}^{00}_{j}-({}^{i0}_{j}+{}^{j0}_{i}-{}^{ij}_{0}){}^{00}_{0}), \tag{94} $$
$$ P_{ij(000n)}=\tfrac14\, g^{00}g^{n0}({}^{00}_{i}({}^{jn}_{0}+{}^{n0}_{j}-{}^{j0}_{n}) -({}^{i0}_{j}+{}^{j0}_{i}-{}^{ij}_{0})(2\,{}^{n0}_{0}-{}^{00}_{n})), \tag{95} $$
$$ P_{ij(00m0)}=\tfrac14\, g^{00}g^{m0}(({}^{im}_{0}+{}^{m0}_{i}-{}^{i0}_{m}){}^{00}_{j} -({}^{im}_{j}+{}^{jm}_{i}-{}^{ij}_{m}){}^{00}_{0}), \tag{96} $$
$$ P_{ij(00mn)}=\tfrac14\, g^{00}g^{mn}(({}^{im}_{0}+{}^{m0}_{i}-{}^{i0}_{m})({}^{jn}_{0}+{}^{n0}_{j}-{}^{j0}_{n}) -({}^{im}_{j}+{}^{jm}_{i}-{}^{ij}_{m})(2\,{}^{n0}_{0}-{}^{00}_{n})), \tag{97} $$
$$ P_{ij(0l00)}=\tfrac14\, g^{l0}g^{00}({}^{00}_{i}({}^{j0}_{l}+{}^{l0}_{j}-{}^{jl}_{0})-({}^{i0}_{j}+{}^{j0}_{i}-{}^{ij}_{0}){}^{00}_{l}), \tag{98} $$

$$\begin{aligned}P_{ij(0l0n)}=&\tfrac{1}{4}\, g^{l0}g^{n0}({}^{00}_{i}({}^{jn}_{l}+{}^{ln}_{j}-{}^{jl}_{n})\\&-({}^{i0}_{j}+{}^{j0}_{i}-{}^{ij}_{0})({}^{n0}_{l}+{}^{ln}_{0}-{}^{l0}_{n})),\end{aligned}\tag{99}$$

$$\begin{aligned}P_{ij(0lm0)}=&\tfrac{1}{4}\, g^{l0}g^{m0}(({}^{im}_{0}+{}^{m0}_{i}-{}^{i0}_{m})({}^{j0}_{l}+{}^{l0}_{j}-{}^{jl}_{0})\\&-({}^{im}_{j}+{}^{jm}_{i}-{}^{ij}_{m})^{00}_{l}),\end{aligned}\tag{100}$$

$$\begin{aligned}P_{ij(0lmn)}=&\tfrac{1}{4}\, g^{l0}g^{mn}(({}^{im}_{0}+{}^{m0}_{i}-{}^{i0}_{m})({}^{jn}_{l}+{}^{ln}_{j}-{}^{jl}_{n})\\&-({}^{im}_{j}+{}^{jm}_{i}-{}^{ij}_{m})({}^{n0}_{l}+{}^{ln}_{0}-{}^{l0}_{n})),\end{aligned}\tag{101}$$

$$P_{ij(k000)}=\tfrac{1}{4}\, g^{k0}g^{00}(({}^{i0}_{k}+{}^{k0}_{i}-{}^{ik}_{0})^{00}_{j}-({}^{i0}_{j}+{}^{j0}_{i}-{}^{ij}_{0})^{00}_{k}),\tag{102}$$

$$\begin{aligned}P_{ij(k00n)}=&\tfrac{1}{4}\, g^{k0}g^{n0}(({}^{i0}_{k}+{}^{k0}_{i}-{}^{ik}_{0})({}^{jn}_{0}+{}^{n0}_{j}-{}^{j0}_{n})\\&-({}^{i0}_{j}+{}^{j0}_{i}-{}^{ij}_{0})({}^{kn}_{0}+{}^{n0}_{k}-{}^{k0}_{n})),\end{aligned}\tag{103}$$

$$\begin{aligned}P_{ij(k0m0)}=&\tfrac{1}{4}\, g^{k0}g^{m0}(({}^{im}_{k}+{}^{km}_{i}-{}^{ik}_{m})^{00}_{j}\\&-({}^{im}_{j}+{}^{jm}_{i}-{}^{ij}_{m})^{00}_{k}),\end{aligned}\tag{104}$$

$$\begin{aligned}P_{ij(k0mn)}=&\tfrac{1}{4}\, g^{k0}g^{mn}(({}^{im}_{k}+{}^{km}_{i}-{}^{ik}_{m})({}^{jn}_{0}+{}^{n0}_{j}-{}^{j0}_{n})\\&-({}^{im}_{j}+{}^{jm}_{i}-{}^{ij}_{m})({}^{kn}_{0}+{}^{n0}_{k}-{}^{k0}_{n})),\end{aligned}\tag{105}$$

$$\begin{aligned}P_{ij(kl00)}=&\tfrac{1}{4}\, g^{kl}g^{00}(({}^{i0}_{k}+{}^{k0}_{i}-{}^{ik}_{0})({}^{j0}_{l}+{}^{l0}_{j}-{}^{jl}_{0})\\&-({}^{i0}_{j}+{}^{j0}_{i}-{}^{ij}_{0})({}^{k0}_{l}+{}^{l0}_{k}-{}^{kl}_{0})),\end{aligned}\tag{106}$$

$$\begin{aligned}P_{ij(kl0n)}=&\tfrac{1}{4}\, g^{kl}g^{n0}(({}^{i0}_{k}+{}^{k0}_{i}-{}^{ik}_{0})({}^{jn}_{l}+{}^{ln}_{j}-{}^{jl}_{n})\\&-({}^{i0}_{j}+{}^{j0}_{i}-{}^{ij}_{0})({}^{kn}_{l}+{}^{ln}_{k}-{}^{kl}_{n})),\end{aligned}\tag{107}$$

$$\begin{aligned}P_{ij(klm0)}=&\tfrac{1}{4}\, g^{kl}g^{n0}(({}^{im}_{k}+{}^{km}_{i}-{}^{ik}_{m})({}^{j0}_{l}+{}^{l0}_{j}-{}^{jl}_{0})\\&-({}^{im}_{j}+{}^{jm}_{i}-{}^{ij}_{m})({}^{k0}_{l}+{}^{l0}_{k}-{}^{kl}_{0})),\end{aligned}\tag{108}$$

$$\begin{aligned}P_{ij(klmn)}=&\tfrac{1}{4}\, g^{kl}g^{mn}(({}^{im}_{k}+{}^{km}_{i}-{}^{ik}_{m})({}^{jn}_{l}+{}^{ln}_{j}-{}^{jl}_{n})\\&-({}^{im}_{j}+{}^{jm}_{i}-{}^{ij}_{m})({}^{kn}_{l}+{}^{ln}_{k}-{}^{kl}_{n})).\end{aligned}\tag{109}$$