The speed of photons in absolute gravity

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Photons in absolute gravity do not move at a constant absolute speed in absolute space. They change speeds in gravitational fields. In addition, a photon's speed depends on its orientation with respect to the gravitational fields. This paper shows how to calculate the absolute speed of a photon. For a given unit direction vector, there are two possible speeds. In most places in the universe one speed is positive and the other is negative. The two speeds are the solutions to a quadratic equation in terms of the gravitational fields and the unit direction vector.

INTRODUCTION

Photons in absolute gravity[1] do not move at a constant absolute speed in absolute space. They change speeds in gravitational fields. In addition, a photon's speed depends on its orientation with respect to the gravitational fields.

The main result of this note is that a photon's absolute speed v in a given absolute unit direction $\hat{\mathbf{v}}$ in the fields g, $\mathbf{w}$, and $\mathbf{S}$ (using the notation introduced in [2]) is:

$$\frac{v}{c} = \frac{-\mathbf{w} \cdot \hat{\mathbf{v}} \pm \sqrt{(\mathbf{w} \cdot \hat{\mathbf{v}})^2 - g(\mathbf{S} \cdot (\hat{\mathbf{v}}\hat{\mathbf{v}}^\intercal))}}{\mathbf{S} \cdot (\hat{\mathbf{v}}\hat{\mathbf{v}}^\intercal)}. \tag{1}$$

The $\pm$ indicates that photons can have two different speeds for a given unit direction vector. In most places in the universe one speed is positive and the other is negative. For example, consider a laboratory on Earth. In order to maintain constant speeds in all directions relative to the laboratory, photons move faster in the direction of Earth's rotation, and slower in the opposite direction.

DERIVATION

We can begin deriving equation (1) starting from the line element from general relativity, in a metric with signature $(1, -1, -1, -1)$, and proper time measured in seconds:

$$(\mathrm{d}s)^2 = (c\,\mathrm{d}\tau)^2 = g_{\mu\nu}\,\mathrm{d}x_\mu\,\mathrm{d}x_\nu. \tag{2}$$

For a photon, the proper time τ is 0, so for a photon:

$$0 = g_{\mu\nu}\,\mathrm{d}x_\mu\,\mathrm{d}x_\nu. \tag{3}$$

Dividing both sides by $(\mathrm{d}x_0)^2$ gives:

$$0 = g_{\mu\nu}\frac{\mathrm{d}x_\mu}{\mathrm{d}x_0}\frac{\mathrm{d}x_\nu}{\mathrm{d}x_0}. \tag{4}$$

Next, we can expand the sums over μ and ν to separate the cases $\mu = 0$ or $\nu = 0$ from the cases $\mu, \nu = 1, 2, 3$, and then replace μ and ν by i and j for $i, j = 1, 2, 3$. Since $g_{\mu\nu}$ is symmetric, we can coalesce the cases $g_{i0} = g_{0i}$:

$$0 = g_{00}\frac{\mathrm{d}x_0}{\mathrm{d}x_0}\frac{\mathrm{d}x_0}{\mathrm{d}x_0} + 2g_{i0}\frac{\mathrm{d}x_i}{\mathrm{d}x_0}\frac{\mathrm{d}x_0}{\mathrm{d}x_0} + g_{ij}\frac{\mathrm{d}x_i}{\mathrm{d}x_0}\frac{\mathrm{d}x_j}{\mathrm{d}x_0}. \tag{5}$$

Noting that $\frac{\mathrm{d}x_0}{\mathrm{d}x_0} = 1$, we can eliminate the factors of $\frac{\mathrm{d}x_0}{\mathrm{d}x_0}$:

$$0 = g_{00} + 2g_{i0}\frac{\mathrm{d}x_i}{\mathrm{d}x_0} + g_{ij}\frac{\mathrm{d}x_i}{\mathrm{d}x_0}\frac{\mathrm{d}x_j}{\mathrm{d}x_0}. \tag{6}$$

Noting that $\mathrm{d}x_0 = c\,\mathrm{d}t$, we can replace occurrences of $\mathrm{d}x_0$:

$$0 = g_{00} + \frac{2}{c}g_{i0}\frac{\mathrm{d}x_i}{\mathrm{d}t} + \frac{1}{c^2}g_{ij}\frac{\mathrm{d}x_i}{\mathrm{d}t}\frac{\mathrm{d}x_j}{\mathrm{d}t}. \tag{7}$$

Now we can write $\frac{\mathrm{d}x_{i,j}}{\mathrm{d}t}$ as elements $\mathbf{v}_{i,j}$ of the absolute velocity three-vector $\mathbf{v}$:

$$0 = g_{00} + \frac{2}{c} g_{i0} \mathbf{v}_i + \frac{1}{c^2} g_{ij} \mathbf{v}_i \mathbf{v}_j. \tag{8}$$

Recalling from [2] that g, $\mathbf{w}$, and $\mathbf{S}$, are defined in terms of $g_{\mu\nu}$ by:

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} g & \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 \\ \mathbf{w}_1 & \mathbf{S}_{11} & \mathbf{S}_{12} & \mathbf{S}_{13} \\ \mathbf{w}_2 & \mathbf{S}_{21} & \mathbf{S}_{22} & \mathbf{S}_{23} \\ \mathbf{w}_3 & \mathbf{S}_{31} & \mathbf{S}_{32} & \mathbf{S}_{33} \end{bmatrix} = \begin{bmatrix} g & \mathbf{w}^\intercal \\ \mathbf{w} & \mathbf{S} \end{bmatrix}, \tag{9}$$

we can substitute for the $g_{\mu\nu}$ in equation (8) to get:

$$0 = g + \frac{2}{c} \mathbf{w}_i \mathbf{v}_i + \frac{1}{c^2} \mathbf{S}_{ij} \mathbf{v}_i \mathbf{v}_j. \tag{10}$$

We can eliminate the indexes i and j by writing equation (10) in vector/matrix notation:

$$0 = g + \frac{2}{c} (\mathbf{w} \cdot \mathbf{v}) + \frac{1}{c^2} (\mathbf{S} \cdot (\mathbf{v}\mathbf{v}^\intercal)), \tag{11}$$

where the $\cdot$ between $\mathbf{S}$ and $(\mathbf{v}\mathbf{v}^\intercal)$ is the matrix dot product defined in [2].

Next we can write $\mathbf{v}$ as $v\,\hat{\mathbf{v}}$ and separate out v:

$$0 = g + \frac{2}{c} (\mathbf{w} \cdot \hat{\mathbf{v}}) v + \frac{1}{c^2} (\mathbf{S} \cdot (\hat{\mathbf{v}}\hat{\mathbf{v}}^\intercal)) v^2. \tag{12}$$

Solving the quadratic equation for v gives:

$$v = \frac{-\frac{2}{c}(\mathbf{w} \cdot \hat{\mathbf{v}}) \pm \sqrt{\frac{4}{c^2}(\mathbf{w} \cdot \hat{\mathbf{v}})^2 - \frac{4g}{c^2}(\mathbf{S} \cdot (\hat{\mathbf{v}}\hat{\mathbf{v}}^\intercal))}}{\frac{2}{c^2}(\mathbf{S} \cdot (\hat{\mathbf{v}}\hat{\mathbf{v}}^\intercal))}. \tag{13}$$

Multiplying both sides by $\frac{1}{c}$ and then canceling out constants gives:

$$\frac{v}{c} = \frac{-\mathbf{w} \cdot \hat{\mathbf{v}} \pm \sqrt{(\mathbf{w} \cdot \hat{\mathbf{v}})^2 - g(\mathbf{S} \cdot (\hat{\mathbf{v}}\hat{\mathbf{v}}^\intercal))}}{\mathbf{S} \cdot (\hat{\mathbf{v}}\hat{\mathbf{v}}^\intercal)}, \tag{14}$$

which is equation (1).

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[1] Parker, D. B., "General Relativity in Absolute Space and Time", 2022, preprint, https://pgu.org
[2] Parker, D. B., "The absolute gravity force equation as classical mechanics", 2023, preprint, https://pgu.org