Rate of change of proper time with respect to absolute time

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In order to correct the proper clocks of relativity to create absolute clocks, it is useful to know the rate of change of proper time with respect to absolute time. This note presents a formula for the differential rate, written using the three-dimensional vector calculus notation of absolute gravity. The result depends both on the gravitational fields and on the absolute velocity of the proper clock.

INTRODUCTION

In order to correct the proper clocks of relativity to create absolute clocks[1], it is useful to know the rate of change of proper time with respect to absolute time. This note presents a formula for the differential rate, written using the three-dimensional vector calculus notation of absolute gravity[2].

The main result of this note is that the rate of change of proper time τ with respect to absolute time t for a proper clock moving with absolute velocity $\mathbf{v}$ in the fields g, $\mathbf{w}$, and $\mathbf{S}$ is:

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \sqrt{g + \frac{2}{c}(\mathbf{w} \cdot \mathbf{v}) + \frac{1}{c^2}(\mathbf{S} \cdot (\mathbf{v}\mathbf{v}^{\intercal}))}, \tag{1}$$

remembering that we may need to change the sign of the quantity under the square root sign if we have crossed a singularity that changes the metric signature.

DERIVATION

This derivation has much in common with the derivation of the absolute speed of photons in [3].

We can begin deriving equation (1) starting from the formula for proper time from general relativity, with a metric signature of $(1, -1, -1, -1)$, and proper time τ measured in seconds. The metric signature of $(1, -1, -1, -1)$ ensures that proper time τ is positive as we move into the future:

$$(c\,\mathrm{d}\tau)^2 = g_{\mu\nu}\,\mathrm{d}x_\mu\,\mathrm{d}x_\nu. \tag{2}$$

Dividing both sides by $(\mathrm{d}x_0)^2$ gives:

$$c^2\left(\frac{\mathrm{d}\tau}{\mathrm{d}x_0}\right)^2 = g_{\mu\nu}\frac{\mathrm{d}x_\mu}{\mathrm{d}x_0}\frac{\mathrm{d}x_\nu}{\mathrm{d}x_0}. \tag{3}$$

We can expand the sums over μ and ν to separate the cases $\mu = 0$ or $\nu = 0$ from the cases $\mu, \nu = 1, 2, 3$, and then replace μ and ν by i and j for $i, j = 1, 2, 3$. Since $g_{\mu\nu}$ is symmetric, we can coalesce the cases $g_{i0} = g_{0i}$:

$$c^2\left(\frac{\mathrm{d}\tau}{\mathrm{d}x_0}\right)^2 = g_{00}\frac{\mathrm{d}x_0}{\mathrm{d}x_0}\frac{\mathrm{d}x_0}{\mathrm{d}x_0} + 2g_{i0}\frac{\mathrm{d}x_i}{\mathrm{d}x_0}\frac{\mathrm{d}x_0}{\mathrm{d}x_0} + g_{ij}\frac{\mathrm{d}x_i}{\mathrm{d}x_0}\frac{\mathrm{d}x_j}{\mathrm{d}x_0}. \tag{4}$$

Noting that $\frac{\mathrm{d}x_0}{\mathrm{d}x_0} = 1$, we can eliminate the factors of $\frac{\mathrm{d}x_0}{\mathrm{d}x_0}$:

$$c^2\left(\frac{\mathrm{d}\tau}{\mathrm{d}x_0}\right)^2 = g_{00} + 2g_{i0}\frac{\mathrm{d}x_i}{\mathrm{d}x_0} + g_{ij}\frac{\mathrm{d}x_i}{\mathrm{d}x_0}\frac{\mathrm{d}x_j}{\mathrm{d}x_0}. \tag{5}$$

Noting that $\mathrm{d}x_0 = c\,\mathrm{d}t$, we can replace occurrences of $\mathrm{d}x_0$:

$$\left(\frac{\mathrm{d}\tau}{\mathrm{d}t}\right)^2 = g_{00} + \frac{2}{c}g_{i0}\frac{\mathrm{d}x_i}{\mathrm{d}t} + \frac{1}{c^2}g_{ij}\frac{\mathrm{d}x_i}{\mathrm{d}t}\frac{\mathrm{d}x_j}{\mathrm{d}t}. \tag{6}$$

Now we can write $\frac{\mathrm{d}x_{i,j}}{\mathrm{d}t}$ as elements $\mathbf{v}_{i,j}$ of the absolute velocity three-vector $\mathbf{v}$:

$$\left(\frac{\mathrm{d}\tau}{\mathrm{d}t}\right)^2 = g_{00} + \frac{2}{c}g_{i0}\mathbf{v}_i + \frac{1}{c^2}g_{ij}\mathbf{v}_i\mathbf{v}_j. \tag{7}$$

Recalling from [2] that g, $\mathbf{w}$, and $\mathbf{S}$, are defined in terms of $g_{\mu\nu}$ by:

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} g & \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 \\ \mathbf{w}_1 & \mathbf{S}_{11} & \mathbf{S}_{12} & \mathbf{S}_{13} \\ \mathbf{w}_2 & \mathbf{S}_{21} & \mathbf{S}_{22} & \mathbf{S}_{23} \\ \mathbf{w}_3 & \mathbf{S}_{31} & \mathbf{S}_{32} & \mathbf{S}_{33} \end{bmatrix} = \begin{bmatrix} g & \mathbf{w}^\intercal \\ \mathbf{w} & \mathbf{S} \end{bmatrix}, \tag{8}$$

we can substitute for the $g_{\mu\nu}$ in equation (7) to get:

$$\left(\frac{\mathrm{d}\tau}{\mathrm{d}t}\right)^2 = g + \frac{2}{c}\mathbf{w}_i\mathbf{v}_i + \frac{1}{c^2}\mathbf{S}_{ij}\mathbf{v}_i\mathbf{v}_j. \tag{9}$$

We can eliminate the indexes i and j by writing equation (9) in vector/matrix notation:

$$\left(\frac{\mathrm{d}\tau}{\mathrm{d}t}\right)^2 = g + \frac{2}{c}(\mathbf{w}\cdot\mathbf{v}) + \frac{1}{c^2}(\mathbf{S}\cdot(\mathbf{v}\mathbf{v}^\intercal)), \tag{10}$$

where the $\cdot$ between $\mathbf{S}$ and $(\mathbf{v}\mathbf{v}^\intercal)$ is the matrix dot product defined in [2].

Proper time τ increases as we move into the future because we used the metric signature $(1,-1,-1,-1)$ for equation (2). Absolute time also increases as we move into the future. Hence $\frac{\mathrm{d}\tau}{\mathrm{d}t}$ is positive, and we are free to take the positive square root of both sides of equation (10) to get:

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \sqrt{g + \frac{2}{c}(\mathbf{w}\cdot\mathbf{v}) + \frac{1}{c^2}(\mathbf{S}\cdot(\mathbf{v}\mathbf{v}^\intercal))}, \tag{11}$$

which is equation (1). However, we may need to change the sign of the quantity under the square root sign if we have crossed a singularity that changes the metric signature.

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[1] Parker, D. B., "On absolute clocks and rulers", 2023, preprint, https://pgu.org
[2] Parker, D. B., "The absolute gravity force equation as classical mechanics", 2023, preprint, https://pgu.org
[3] Parker, D. B., "The speed of photons in absolute gravity", 2023, preprint, https://pgu.org